2. Work and Energy

> ➤ **Work**   ➤ **Energy**   ➤ **Mechanical energy**
> ➤ **Law of conservation of energy**   ➤ **Free fall**

Observe
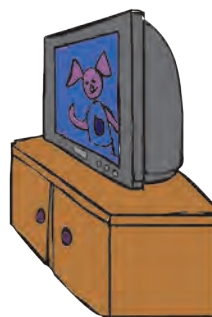
2.1 Various activities

Can you tell?
1. In which of the pictures above has work been done?
2. From scientific point of view, when do we say that no work was done?

Generally, any mental or physical activity is referred to as work. When we walk or run, the energy in our body is used to do the necessary work. We say that a girl who is studying is working or performing work. But that is mental work.

In physics, we deal with physical work. Work has a special meaning in physics.

'Work is said to be done when a force applied on an object causes displacement of the object.'

You have already learnt that the work done by a force acting on an object is the product of the magnitude of the force and the displacement of the object in the direction of the force. Thus,   Work = force × displacement

Can you recall?   What are different types of forces and their examples?

Minakshee wants to displace a wooden block from point A to point B along the surface of a table as shown in figure 2.2A. She has used force F for the purpose. Has all the energy she spent been used to produce acceleration in the block? Which forces have been overcome using that energy?

Use your brain power !

You have learnt how to calculate the work done on an object when the displacement is in the direction of the applied force. But if the displacement is not in the direction of the applied force, how do we calculate the amount of work done?
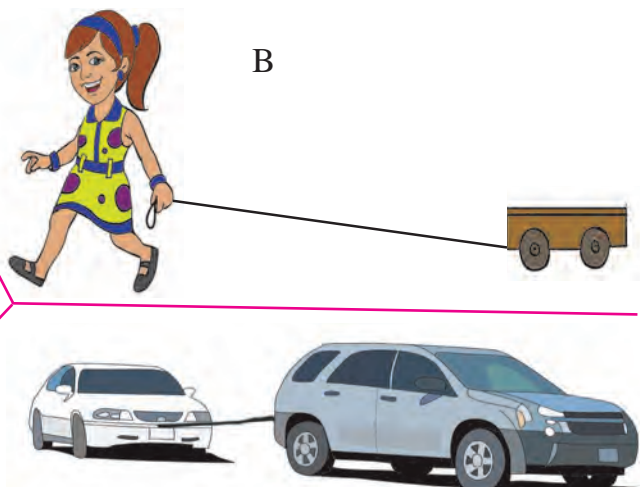
2.2 Displacement of an object

You must have seen the events depicted in the pictures B and C above. When a child pulls a toy with the help of a string, the direction of the force is different from that of displacement. Similarly, when a large vehicle tows a small one, the directions of force and the displacements are different. In both these cases, the direction of force makes an angle with the direction of displacement. Let us see how to calculate work done in such cases.

When a child pulls a toy cart, force is applied along the direction of the string while the cart is pulled along the horizontal surface. In this case, in order to calculate the amount of work done, we have to convert the applied force into the force acting along the direction of displacement.

Let F be the applied force and F_1 be its component in the direction of displacement. Let s be the displacement. The amount of work done is given by

$W = F_1.s$ ……………………………… (1)

The force F is applied in the direction of the string i. e. at an angle with the horizontal. Let θ be the angle that the string makes with the horizontal. We can determine the component F_1, of this force F, which acts in the horizontal direction by means of trigonometry.

2.3 Force used for the displacement

(see figure2.3)

$\cos θ = $ base / hypotenuse

$\cos θ = F_1 / F$
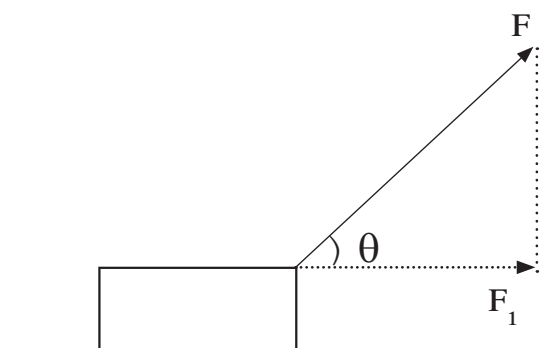
$F_1 = F \cos θ$

Thus, the work done by F_1 is

$W = F \cos θ \ s$

$W = F s \cos θ$

Enter your conclusions about the work done for the specific values of θ in the following table.

θ	cos θ	W = F s cos θ	Conclusion
0^0	1	W = F s	
90^0	0	0	
180^0	−1	W = −F s	

Unit of work

Work = Force × Displacement

In SI system, the unit of force is newton (N) and the unit of displacement is metre (m). Thus, the unit of force is newton-metre. This is called joule.

1 Joule : If a force of 1 newton displaces an object through 1 metre in the direction of the force, the amount of work done on the object is 1 joule.

∴ 1 joule = 1 newton × 1 metre

$1 \text{ J} = 1 \text{ N} \times 1 \text{ m}$

In CGS system, the unit of force is dyne and that of displacement is centimeter (cm). Thus, the unit of work done is dyne-centimetre. This is called an erg.

1 erg : If a force of 1 dyne displaces an object through 1 centimetre in the direction of the force, the amount of work done is 1 erg.

1 erg = 1 dyne × 1 cm

Relationship between joule and erg

We know that, 1 newton = 10^5 dyne and 1 m = 10^2 cm

Work = force × displacement

1 joule = 1 newton × 1 m

1 joule = 10^5 dyne × 10^2 cm

= 10^7 dyne cm

1 joule = 10^7 erg

Positive, negative and zero work

Think before you answer   Discuss the directions of force and of displacement in each of the following cases.

1. Pushing a stalled vehicle
2. Catching the ball which your friend has thrown towards you.
3. Tying a stone to one end of a string and swinging it round and round by the other end of the string.
4. Walking up and down a staircase; climbing a tree.
5. Stopping a moving car by applying brakes.

You will notice that in some of the above examples, the direction of the force and displacement are the same. In some other cases, these directions are opposite to each other, while in some cases, they are perpendicular to each other. In these cases, the work done by the force is as follows.

1. When the force and the displacement are in the same direction ($\theta = 0^0$), the work done by the force is positive.
2. When the force and the displacement are in opposite directions ($\theta = 180^0$), the work done by the force is negative.
3. When the applied force does not cause any displacement or when the force and the displacement are perpendicular to each other ($\theta = 90^0$), the work done by the force is zero.

Take a plastic cup and make a hole in the centre of its bottom. Take a long thread, double it and pass it through the hole. Tie a thick enough knot at the end so that the knot will not pass through the hole, taking care that the two loose ends are below the bottom of the cup. Tie a nut each to the two ends as shown in figure 2.4. Now do the following.

As shown in figure 'A', put the cup on a table, keep one of the nuts in the cup and let the thread carrying the other nut hang down along the side of the table. What happens?

As shown in figure 'B', when the cup is sliding along the table, stop it by putting a ruler in its path.

As shown in figure 'C', keep the cup at the centre of the table and leave the two nuts hanging on opposite sides of the table.

Questions:

1. Figure A - Why does the cup get pulled?
2. Figure B - What is the relation between the displacement of the cup and the force applied through the ruler?
3. In figure C - Why doesn't the cup get displaced?
4. What is the type of work done in figures A, B and C?

In the three actions above, what is the relationship between the applied force and the displacement?
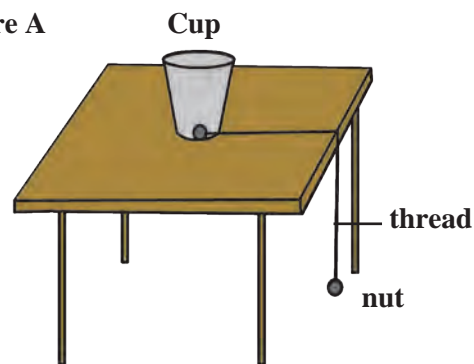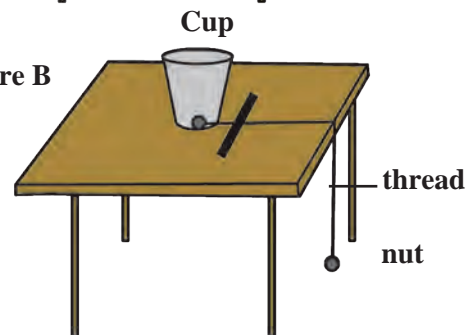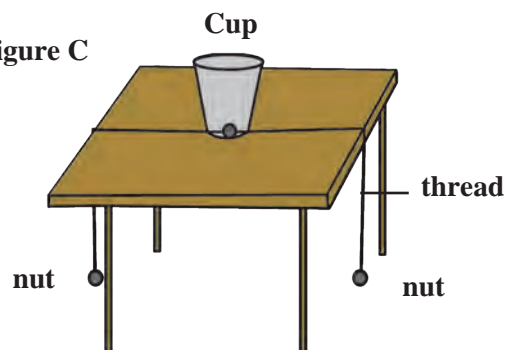
Figure A

Cup

thread

nut

Figure B

Cup

thread

nut

Figure C

Cup

thread

nut

nut

2.4 Positive, negative and zero work

Suppose an artificial satellite is moving around the earth in a circular orbit. As the gravitational force acting on the satellite (along the radius of the circle) and its displacement (along the tangent to the circle) are perpendicular to each other, the work done by the gravitational force is zero

Institutes at work

The National Physical Laboratory, New Delhi, was conceptualized in 1943. It functions under the Council of Scientific and Industrial Research. Its conducts basic research in the various branches of physics and helps various industries and institutes engaged in developmental work. Its main objective is to establish national standards of various physical quantities.

Solved examples

Example 1: Calculate the work done to take an object of mass 20 kg to a height of 10 m.

$(g = 9.8 \text{ m/s}^2)$

Given: m = 20 kg; s = 10 m

g = 9.8 m/s²

$\therefore$ F = m.g

= 20 × (−9.8)

(The negative sign is taken because the displacement is opposite to the direction of the force.)

F = −196 N

$\therefore$ W = F s

= −196 × 10

W = −1960 J

(The negative sign appears because the direction of force is opposite to the direction of displacement so that the work done is negative.)

Example 2 : Pravin has applied a force of 100 N on an object, at an angle of 60^0 to the horizontal. The object gets displaced in the horizontal direction and 400 J work is done. What is the displacement of the object?

$(\cos 60^0 = \dfrac{1}{2})$

Given :

$\theta = 60^0$

F = 100 N

W = 400 J , s = ?

W = F s Cos θ

$400 = 100 \times s \times \dfrac{1}{2}$

$\dfrac{400}{100} = \dfrac{1}{2} \times s$

4 × 2 = s

$\therefore$ s = 8 m

The object will be displaced through 8 m.

Energy

Why does it happen?
1. If a pot having a plant is kept in the dark, the plant languishes.
2. On increasing the volume of a music system or TV beyond a limit, the vessels in the house start vibrating.
3. Collecting sunlight on a paper with the help of a convex lens burns the paper.

The capacity of a body to perform work is called its energy. The units of work and energy are the same. The unit in SI system is **joule** while that in cgs system is **erg**.

You have learnt that energy exists in various forms like mechanical, heat, light, sound, electro-magnetic, chemical, nuclear and solar. In this chapter, we are going to study two forms of mechanical energy, namely, potential energy and kinetic energy.

Kinetic energy

What will happen in the following cases?

1. A fast cricket ball strikes the stumps.
2. The striker hits a coin on the carom board.
3. One marble strikes another in a game of marbles.

From the above examples we understand that when a moving object strikes a stationary object, the stationary object moves. Thus, the moving object has some energy, part or all of which it shares with the stationary object, thereby setting it in motion. 'The energy which an object has because of its motion is called its kinetic energy'. The work done by a force to displace a stationary object through a distance s is the kinetic energy gained by the object.

Kinetic energy = work done on the object

$\therefore$ K.E. = F × s

Expression for kinetic energy :

Suppose a stationary object of mass m moves because of an applied force. Let u be its initial velocity (here u = 0). Let the applied force be F. This generates an acceleration a in the object, and, after time t, the velocity of the object becomes equal to v. The displacement during this time is s. The work done on the object, W = F . s

$$W = F \times s$$

According to Newton's second law of motion,

$$F = ma \text{------- (1)}$$ Similarly, using Newton's second equation of motion

$$s = ut + \frac{1}{2} at^2 \quad \text{However, as initial velocity is zero, u = 0.}$$

$$s = 0 + \frac{1}{2} at^2$$

$$s = \frac{1}{2} at^2 \text{------(2)}$$

$$\therefore W = ma \times \frac{1}{2} at^2 \text{------ using equations (1) and (2)}$$

$$W = \frac{1}{2} m(at)^2 \text{-------(3)}$$

Using Newton's first equation of motion ....

$$v = u + at$$

$$\therefore v = 0 + at$$

$$\therefore v = at$$

$$\therefore v^2 = (at)^2 \text{------(4)}$$

$$\therefore W = \frac{1}{2} mv^2 \text{------- using equations (3) and (4)}$$

The kinetic energy gained by an object is the amount of work done on the object.

$$\therefore K. E. = W$$

$$\therefore K. E. = \frac{1}{2} mv^2$$

Example : A stone having a mass of 250 gm is falling from a height. How much kinetic energy does it have at the moment when its velocity is 2 m/s?

Given : m = 250 g   m = 0.25 kg

v = 2 m/s

$$K.E. = \frac{1}{2} mv^2 = \frac{1}{2} \times 0.25 \times (2)^2 = 0.5 \text{ J}$$

Use your brain power ! The mass of a moving body is doubled, how many times will the kinetic energy increase?

Potential energy

1. An arrow is released from a stretched bow.
2. Water kept at a height flows through a pipe into the tap below.
3. A compressed spring is released.

Which words describe the state of the object in the above examples? Where did the energy required to cause the motion of objects come from?

If the objects were not brought in those states, would they have moved?

'The energy stored in an object because of its specific state or position is called its potential energy.'

1. Hold a chalk at a height of 5 cm from the floor and release it.

2. Now stand up straight and then release the chalk.

3. Is there a difference in the results of the two activities? If so, why?

Expression for potential energy

To carry an object of mass 'm' to a height 'h' above the earth's surface, a force equal to 'mg' has to be used against the direction of the gravitational force. The amount of work done can be calculated as follows.

Work = force x displacement

$W = mg \times h$

$\therefore W = mgh$

$\therefore$ The amount of potential energy stored in the object because of its displacement P.E. = mgh   (W = P.E.)

$\therefore$ Displacement to height h causes energy equal to mgh to be stored in the object.

Example : 500 kg water is stored in the overhead tank of a 10 m high building. Calculate the amount of potential energy stored in the water.

Given :

h = 10 m, m = 500 kg   g = 9.8 m/s^2

$\therefore$ P.E. = mgh

$= 10 \times 9.8 \times 500$

P.E. = 49000 J

Ajay and Atul have been asked to determine the potential energy of a ball of mass m kept on a table as shown in the figure. What answers will they get? Will they be different? What do you conclude from this?

Potential energy is relative. The heights of the ball with respect to Ajay and Atul are different. So the potential energy with respect to them will be different.

Atul

Ajay

h_2

h_1
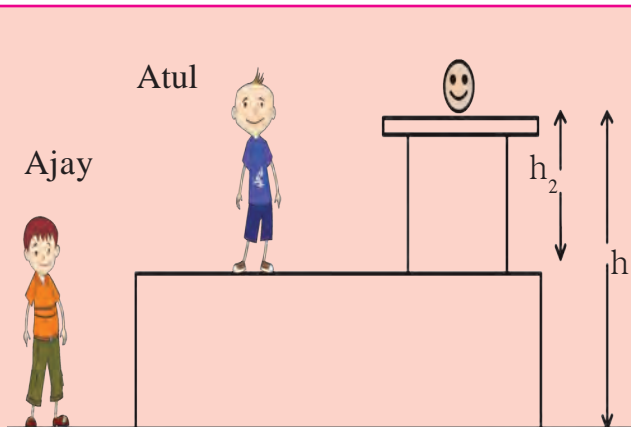
Transformation of energy

Can you tell? Which are the different forms of energy? Which type of energy is used in each of the following processes?

1. A stretched piece of rubber 2. Fast moving car 3. The whistling of a cooker due to steam 4. The crackers exploded in Diwali 5. A fan running on electricity 6. Drawing out pieces of iron from garbage, using a magnet 7. Breaking of a glass window pane because of a loud noise.

Energy can be transformed from one type to another. For example, the exploding firecrackers convert the chemical energy stored in them into light, sound and heat energy.
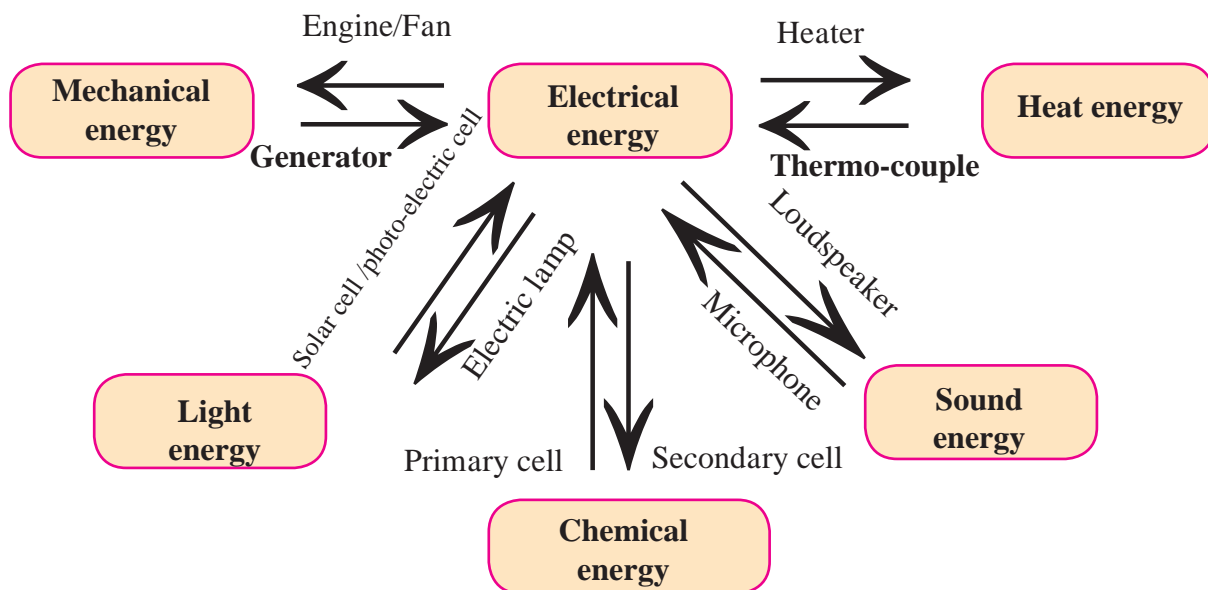
2.5 Tranformation of energy

Observe the above diagram (figure 2.5) and discuss how tranformation of energy takes place, giving example of each.

Law of conservation of energy

'Energy can neither be created nor destroyed. It can be converted from one form into another. Thus, the total amount of energy in the universe remains constant'.

Try this Make two pendulums of the same length with the help of thread and two nuts. Tie another thread in the horizontal position.

Tie the two pendulums to this horizontal thread in such a way that they will not hit each other while swinging. Now swing one of the pendulums and observe. What do you see?

You will see that as the speed of oscillation of the pendulum slowly decreases, the second pendulum which was initially stationary, begins to swing. Thus, one pendulum transfers its energy to the other.
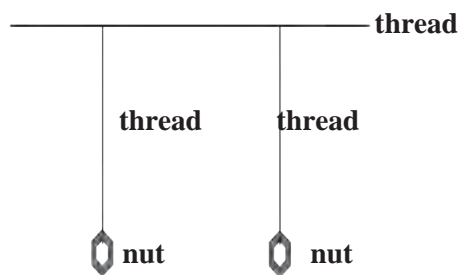
2.6 Coupled oscillators

Free fall

If we release an object from a height, it gets pulled towards the earth because of the gravitational force. An object falling solely under the influence of gravitational force is said to be in free fall or to be falling freely. Let us look at the kinetic and potential energies of an object of mass m, falling freely from height h, when the object is at different heights

As shown in the figure, the point A is at a height h from the ground. Let the point B be at a distance x, vertically below A. Let the point C be on the ground directly below A and B. Let us calculate the energies of the object at A, B and C.

1. When the object is stationary at A, its initial velocity is u = 0

$$\therefore K.E. = \frac{1}{2} \text{ mass x velocity}^2$$

$$= \frac{1}{2} mu^2$$

$$K.E. = 0$$

$$P.E. = mgh$$

$$\therefore \text{Total energy} = K.E. + P.E.$$

$$= 0 + mgh$$

Total Energy = mgh.--- (1)

2. Let the velocity of the object be v_B when it reaches point B, having fallen through a distance x.

u = 0, s = x, a = g

$$v^2 = u^2 + 2as$$

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

$$\therefore K.E. = \frac{1}{2}mv_B^2 = \frac{1}{2}m(2gx)$$

K.E. = mgx

Height of the object when at B = h-x

$$\therefore P.E. = mg(h-x)$$
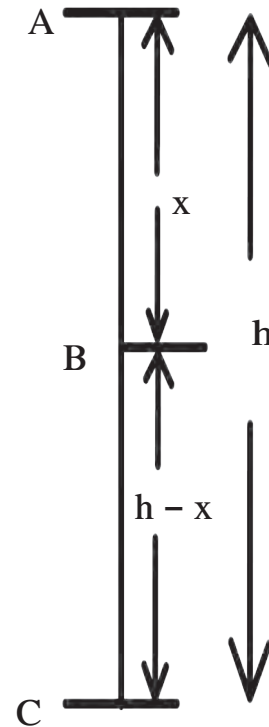
P.E. = mgh − mgx

$$\therefore \text{Total Energy } T.E. = K.E. + P.E.$$

$$= mgx + mgh - mgx$$

$$\therefore T.E. = mgh -------(2)$$

2.7 Free fall

3. Let the velocity of the object be v_C when it reaches the ground, near point C.

u = 0, s = h, a = g

$$v^2 = u^2 + 2as$$

$$v_C^2 = 0 + 2gh$$

$$\therefore K.E. = \frac{1}{2}mv_C^2 = \frac{1}{2}m(2gh)$$

K.E. = mgh

The height of the object from the ground at point C is

h = 0

$$\therefore P.E. = mgh = 0$$

$$\therefore T.E. = K.E. + P.E$$

T.E. = mgh ------(3)

From equations (1), (2) and (3) we see that the total energy of the object is the same at the three points A, B and C.

Thus, every object has potential energy when it is at a height above the ground and it keeps getting converted to kinetic energy as the object falls towards the ground. On reaching the ground (point C), all the potential energy gets converted to kinetic energy. But at any point during the fall the total energy remains constant.

i.e., T.E. = P.E. + K.E.

T.E. at A = mgh + 0 = mgh

T.E. at B = mgx + mg (h - x) = mgh

T.E. at C = 0 + mgh = mgh

Power

Think before you answer   1.   Can your father climb stairs as fast as you can?

2.   Will you fill the overhead water tank with the help of a bucket or an electric motor?
3.   Suppose Rajashree, Yash and Ranjeet have to reach the top of a small hill. Rajashree went by car, Yash went cycling while Ranjeet went walking. If all of them choose the same path, who will reach first and who will reach last?

In the above examples, the work done is the same in each example but the time taken to perform the work is different for each person or each method. The fast or slow rate of the work done is expressed in terms of power. 'Power is the rate at which work is done.'

If W amount of work is done in time t then,

$$\text{Power} = \frac{\text{Work}}{\text{Time}} \quad P = \frac{W}{t}$$

In SI system the unit of work is J, so the unit of power is

J/s. This is called watt

1 watt = 1 joule / 1 second

In the industrial sector the unit used to measure power is called 'horse power.'

1 horse power = 746 watt.

The unit of energy for commercial use is kilo watt hour.
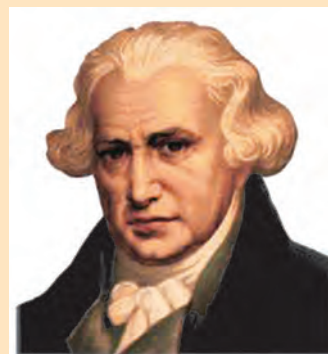
1000 joule work performed in 1 second is 1 kilowatt power.

1 kW h   = 1 kW × 1hr

= 1000 W × 3600 s

= 3600000 J

1 kW h   = 3.6×10^6 J

Electricity used for domestic purposes is measured in units of kilowatt hour.

1 kW h   = 1 unit

An introduction to scientists

The steam engine was invented in 1781 by the Scottish scientist James Watt (1736 – 1819). This invention brought about an industrial revolution. The unit of power is called Watt in his honour. James Watt was the first to use the term 'horse-power'.

Example 1 : Swaralee takes 20 s to carry a bag weighing 20 kg to a height of 5 m. How much power has she used?

Given : m = 20 kg, h = 5 m, t = 40 s

∴ The force which has to be applied by Swaralee,

F = mg = 20 × 9.8

F = 196 N

Work done by Swaralee to carry the bag to a height of 5 m,

W = F s = 196 × 5 = 980 J

$$\therefore \ \text{power} = (P) = \frac{\text{Work}}{t} = \frac{980}{40}$$

P = 24.5 W

Example 2 : A 25 W electric bulb is used for 10 hours every day. How much electricity does it consume each day?

Given :

P = 25, W = 0.025 kW

∴ Energy consumed = power × time

= 0.025 × 10

Energy = 0.25 kW hr

Websites for more information :

www.physicscatalyst.com

www.tryscience.org

Exercises

1. Write detailed answers?

a. Explain the difference between potential energy and kinetic energy.

b. Derive the formula for the kinetic energy of an object of mass m, moving with velocity v.

c. Prove that the kinetic energy of a freely falling object on reaching the ground is nothing but the transformation of its initial potential energy.

d. Determine the amount of work done when an object is displaced at an angle of 30^0 with respect to the direction of the applied force.

e. If an object has 0 momentum, does it have kinetic energy? Explain your answer.

f. Why is the work done on an object moving with uniform circular motion zero?

2. Choose one or more correct alternatives.

a. For work to be performed, energy must be ….
(i) transferred from one place to another (ii) concentrated
(iii) transformed from one type to another (iv) destroyed

b. Joule is the unit of …
(i) force (ii) work (iii) power (iv) energy

c. Which of the forces involved in dragging a heavy object on a smooth, horizontal surface, have the same magnitude?
(i) the horizontal applied force (ii) gravitational force (iii) reaction force in vertical direction (iv) force of friction

d. Power is a measure of the ...……
(i) the rapidity with which work is done (ii) amount of energy required to perform the work (iii) The slowness with which work is performed (iv) length of time

e. While dragging or lifting an object, negative work is done by
(i) the applied force (ii) gravitational force (iii) frictional force (iv) reaction force

3. Rewrite the following sentences using proper alternative.

a. The potential energy of your body is least when you are …..
(i) sitting on a chair (ii) sitting on the ground (iii) sleeping on the ground (iv) standing on the ground

b. The total energy of an object falling freely towards the ground …
(i) decreases (ii) remains unchanged (iii) increases (iv) increases in the beginning and then decreases

c. If we increase the velocity of a car moving on a flat surface to four times its original speed, its potential energy ….
(i) will be twice its original energy (ii) will not change (iii) will be 4 times its original energy (iv) will be 16 times its original energy.

d. The work done on an object does not depend on ….
(i) displacement (ii) applied force (iii) initial velocity of the object (iv) the angle between force and displacement.

4. Study the following activity and answer the questions.

1. Take two aluminium channels of different lengths.

2. Place the lower ends of the channels on the floor and hold their upper ends at the same height.

3. Now take two balls of the same size and weight and release them from the top end of the channels. They will roll down and cover the same distance.

Questions

1. At the moment of releasing the balls, which energy do the balls have?

2. As the balls roll down which energy is converted into which other form of energy?

3. Why do the balls cover the same distance on rolling down?

4. What is the form of the eventual total energy of the balls?

5. Which law related to energy does the above activity demonstrate ? Explain.

5. Solve the following examples.

a. An electric pump has 2 kW power. How much water will the pump lift every minute to a height of 10 m?
(Ans : 1224.5 kg)

b. If a 1200 W electric iron is used daily for 30 minutes, how much total electricity is consumed in the month of April? **(Ans :18 Unit)**

c. If the energy of a ball falling from a height of 10 metres is reduced by 40%, how high will it rebound?
(Ans : 6 m)

d. The velocity of a car increases from 54 km/hr to 72 km/hr. How much is the work done if the mass of the car is 1500 kg ?
(Ans. : 131250 J)

e. Ravi applied a force of 10 N and moved a book 30 cm in the direction of the force. How much was the work done by Ravi?
(Ans: 3 J)

Project :

Study the various instances of trasnformation of energy you see around you and discuss them in class.

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