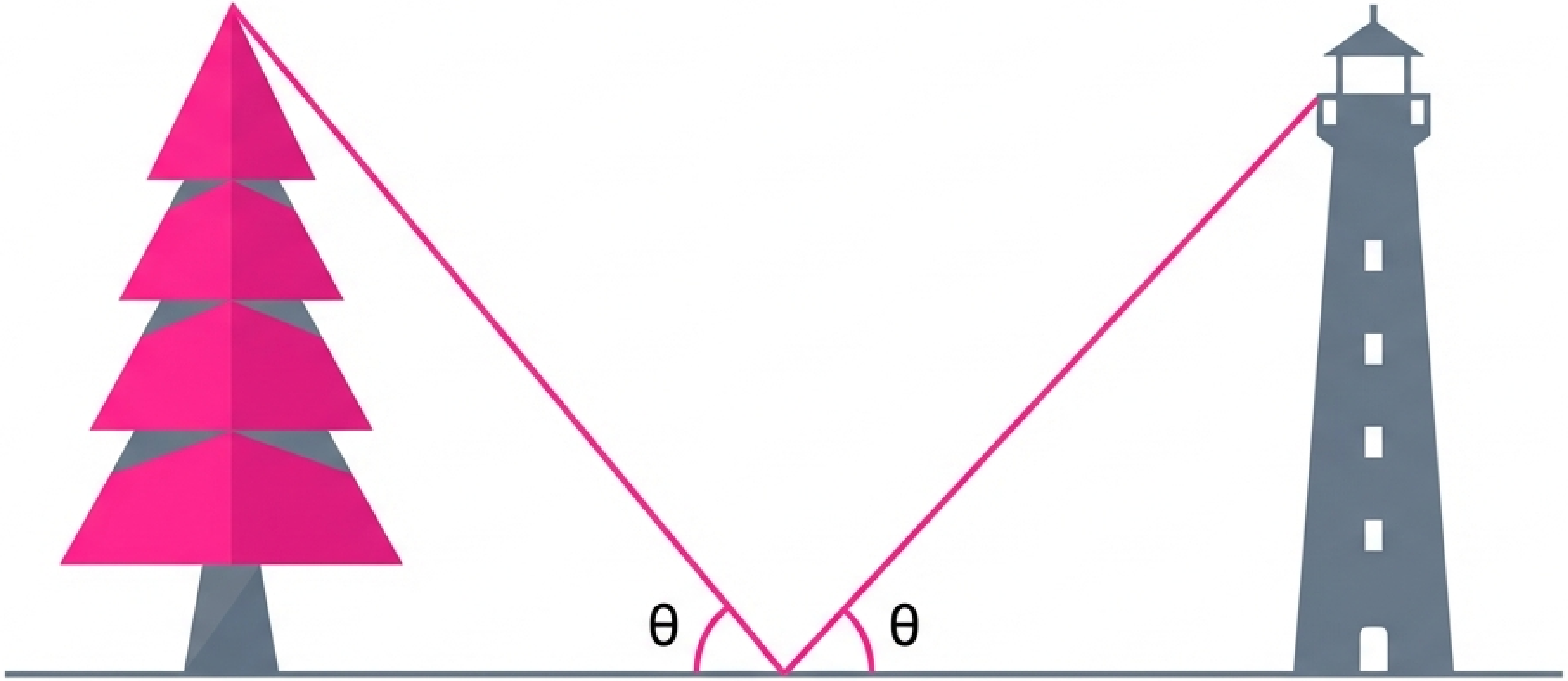
Trigonometry: Measuring the World

From Greek roots (tri-gona-metron) to real-world superpowers.

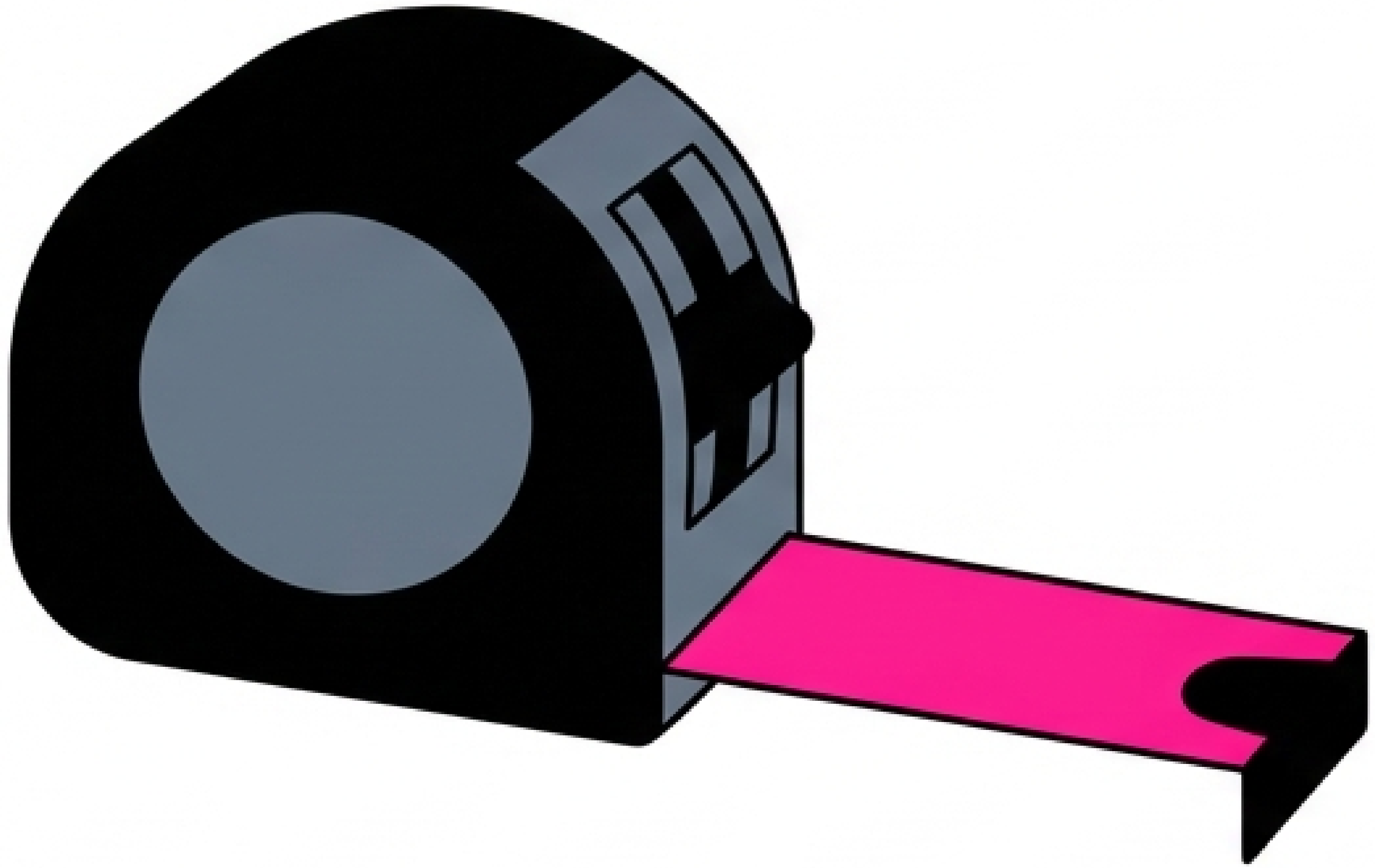


BASED ON CHAPTER 08: INTRODUCTION TO TRIGONOMETRY

The Limitation of Physical Measurement

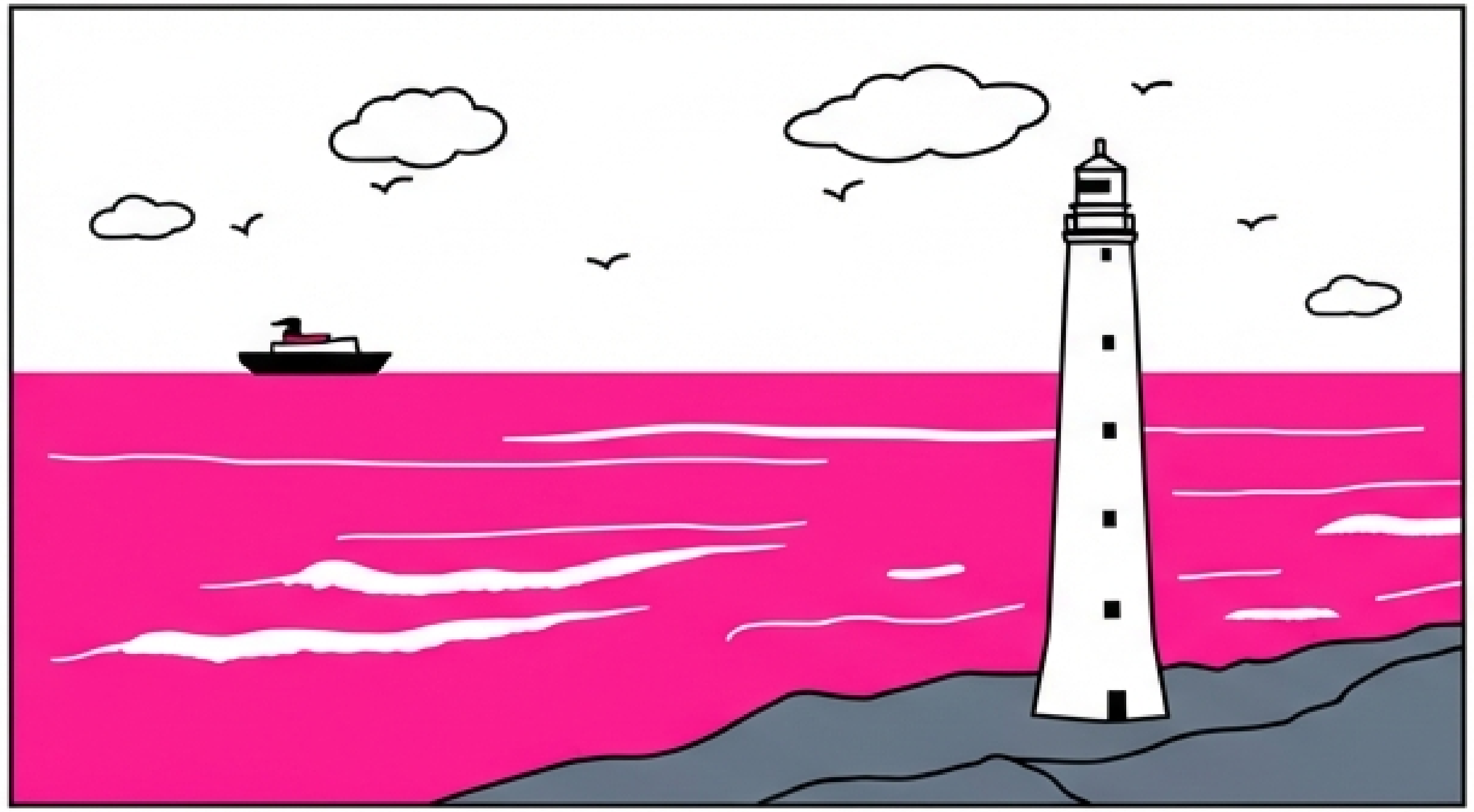
Direct Measurement.

We can measure the ground with a rope.
We can measure a table with a ruler.



Indirect Measurement.

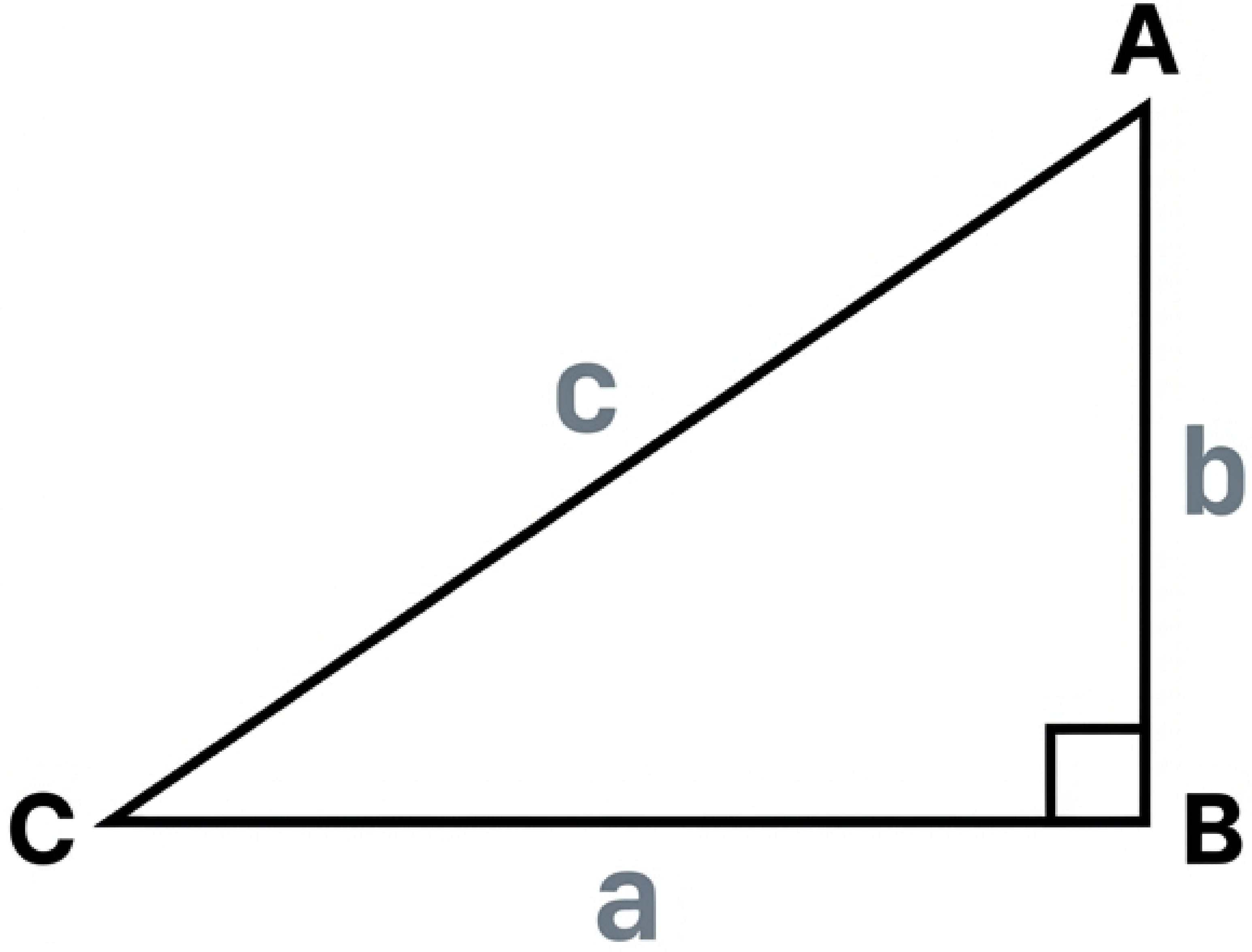
But how do we measure the distance to a ship at sea? How do we measure the height of a towering pine tree without climbing it?



When physical reach ends, mathematics begins.

The Foundation: Right-Angled Triangles

Before looking up, we look at the shape.



1. Pythagoras Theorem

The relationship between sides is static:

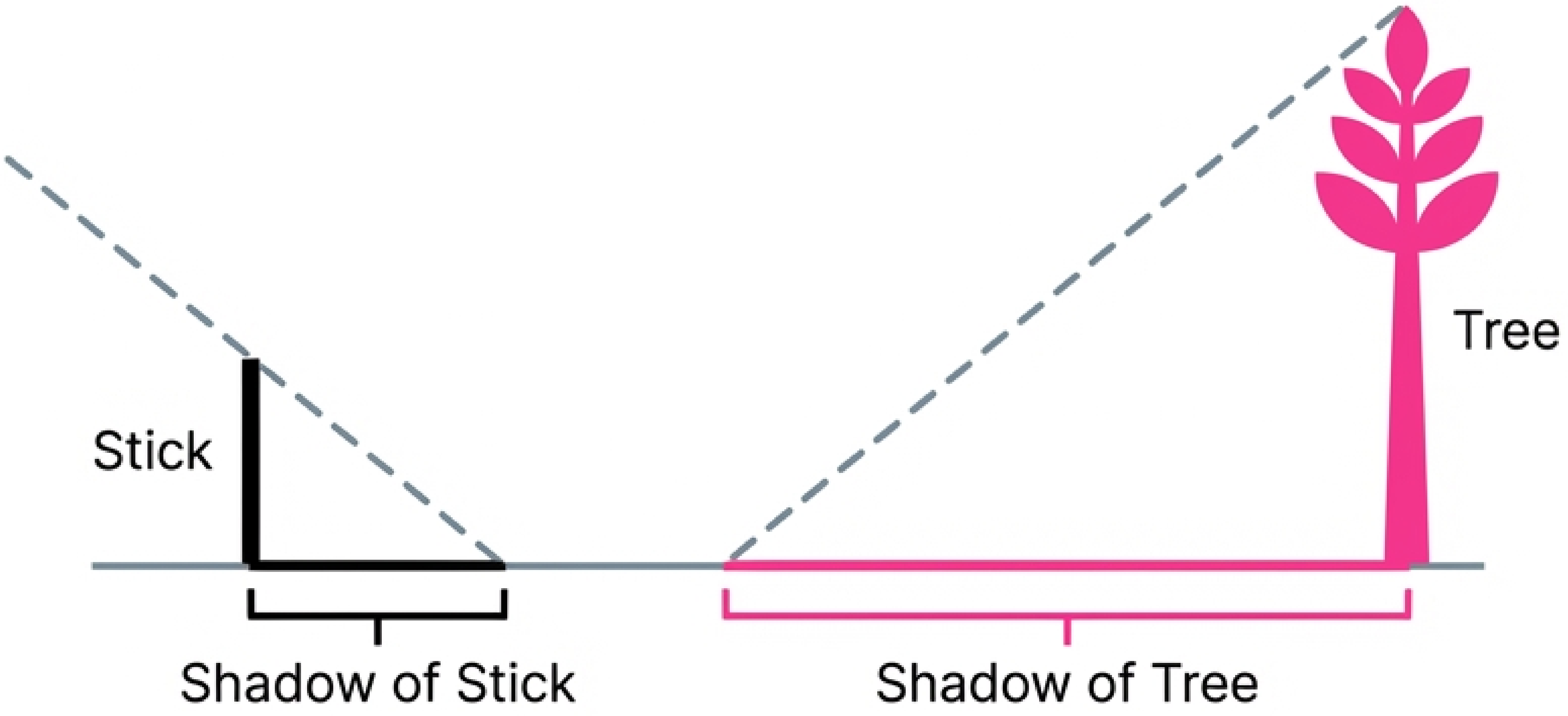
$$a^2 + b^2 = c^2$$

2. Similarity

If triangles are equiangular, they are similar ($\Delta ABC \sim \Delta PQR$). Their sides are always proportional.

$$AB/PQ = BC/QR$$

Measuring with Shadows



1. Sunlight rays are parallel.
2. The stick and its shadow form a small triangle.
3. The tree and its shadow form a large, similar triangle.

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Height of Tree = (Shadow of Tree / Shadow of Stick) × Height of Stick
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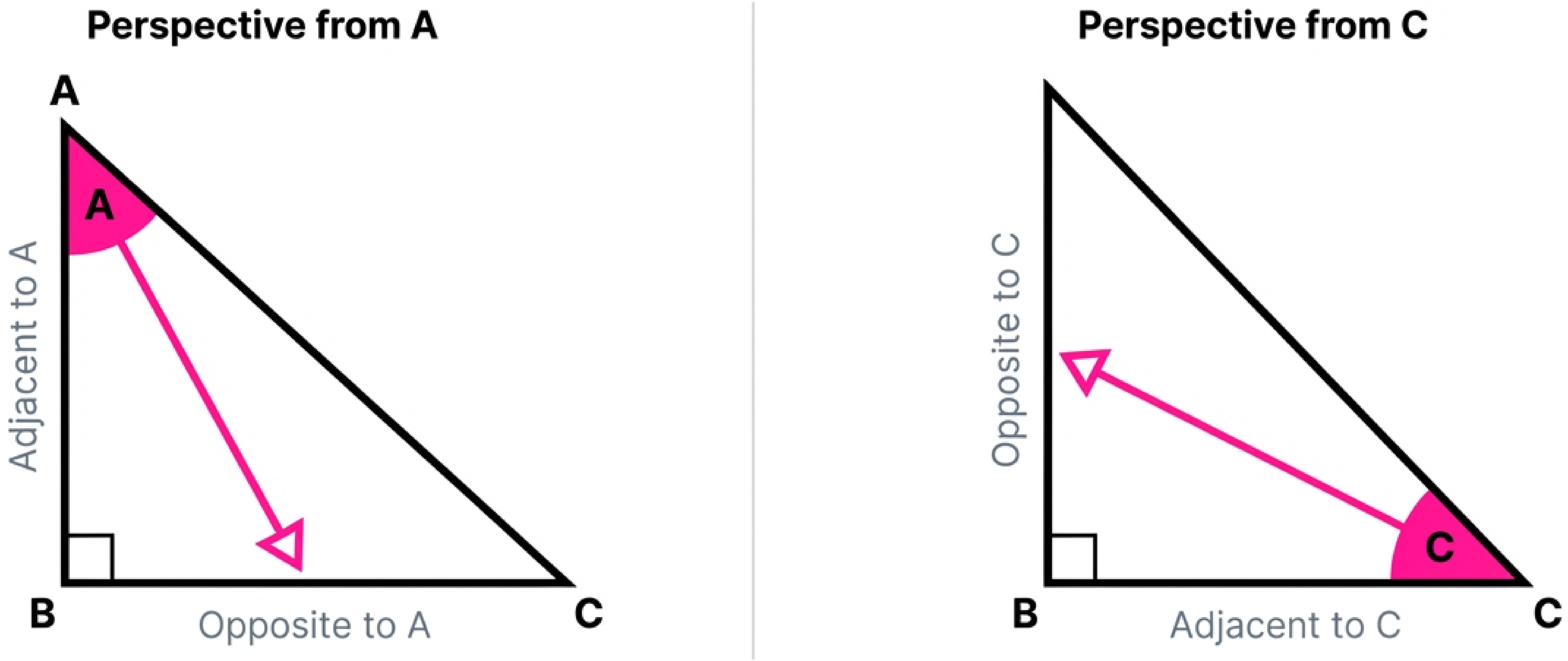
The Vocabulary of Perspective

In trigonometry, “Opposite” and “Adjacent” depend on where you stand.



The Hypotenuse (opposite the 90° angle) never changes.

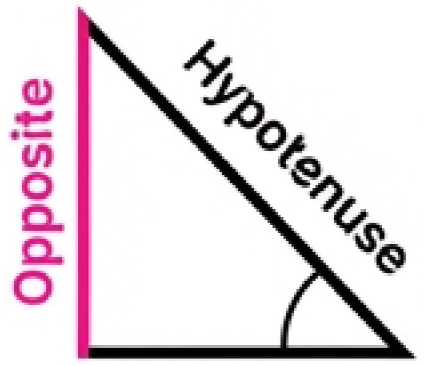
The Three Keys

Defining the Trigonometric Ratios

Sine (sin)

JetBrains Mono

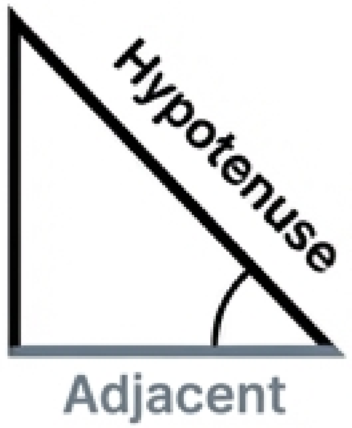
Opposite
Hypotenuse



Cosine (cos)

JetBrains Mono

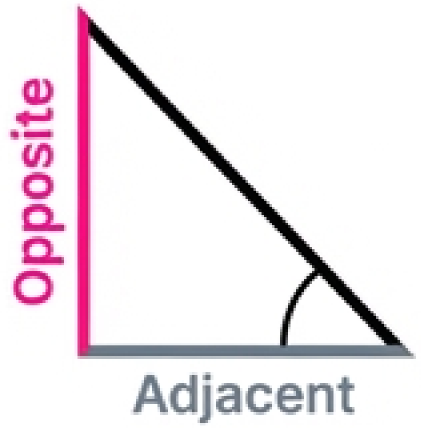
Adjacent
Hypotenuse

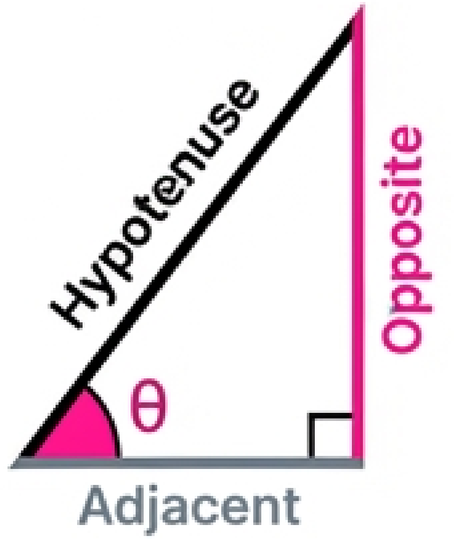


Tangent (tan)

JetBrains Mono

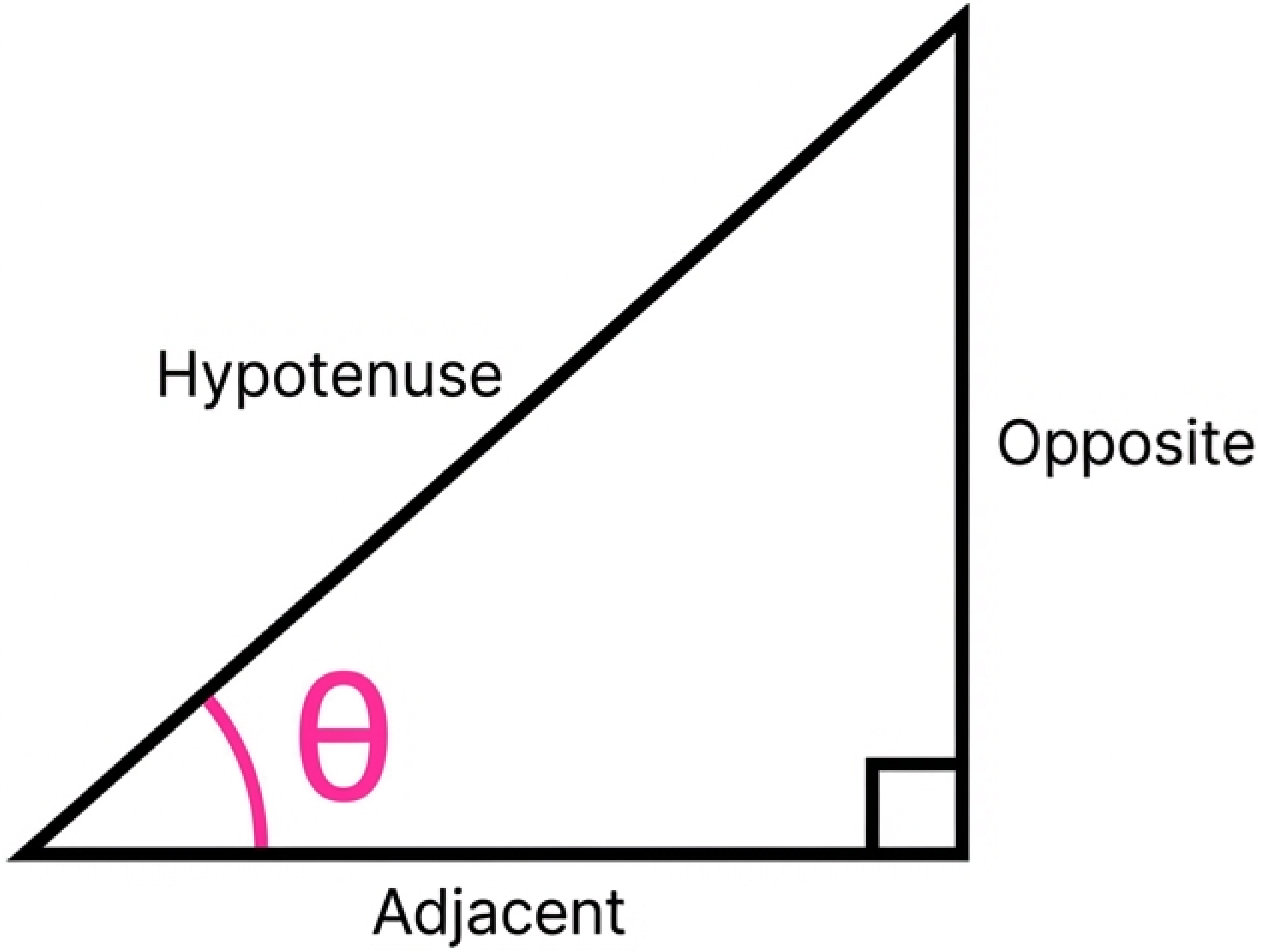
Opposite
Adjacent





Meeting Theta (θ)

Mathematicians use Greek letters to represent unknown angles. The most common is Theta.

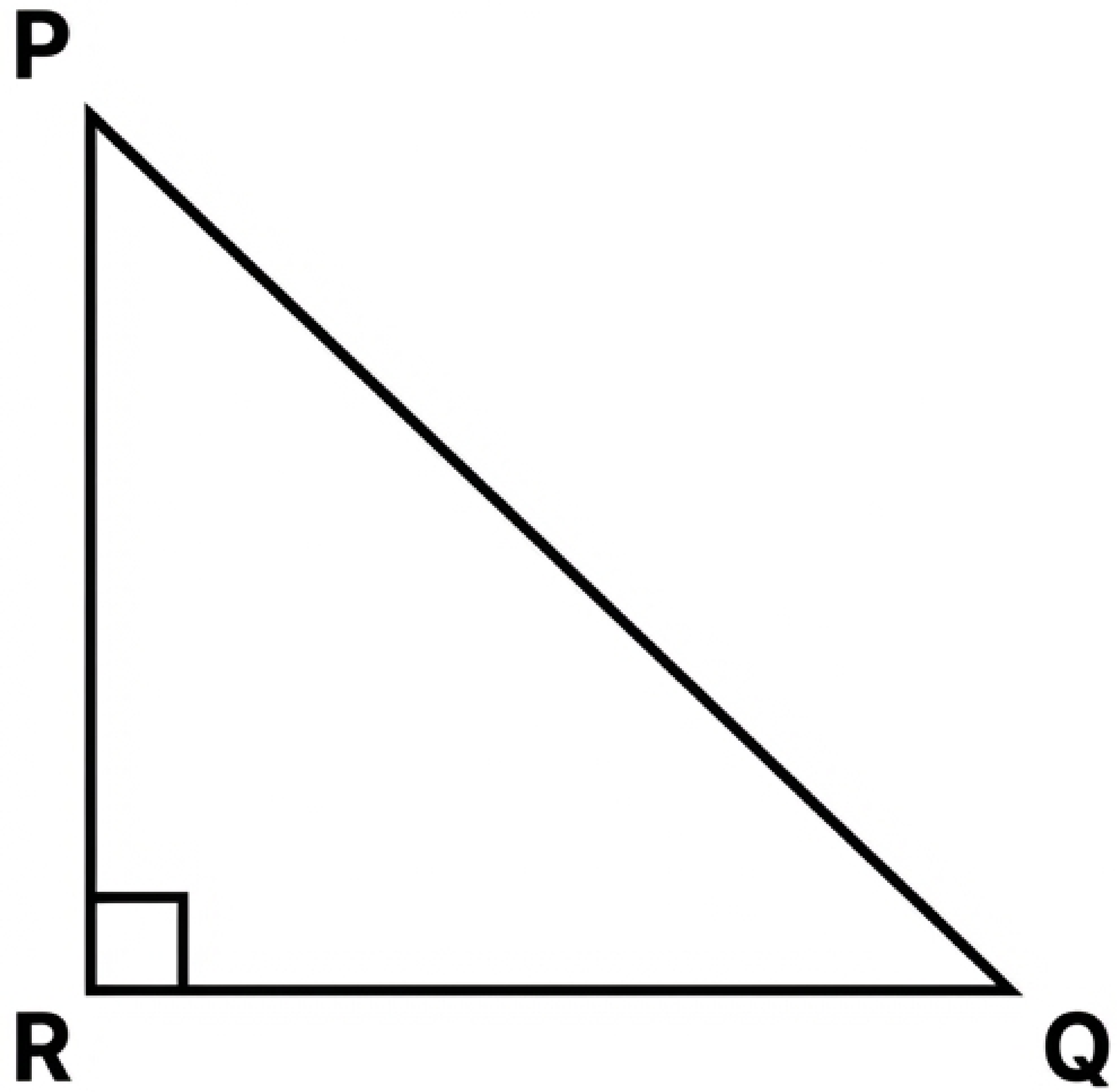


$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

Practice: Defining the Relationships



Find sin P: → QR / PQ

Find cos Q: → QR / PQ

Note: These ratios are identical.

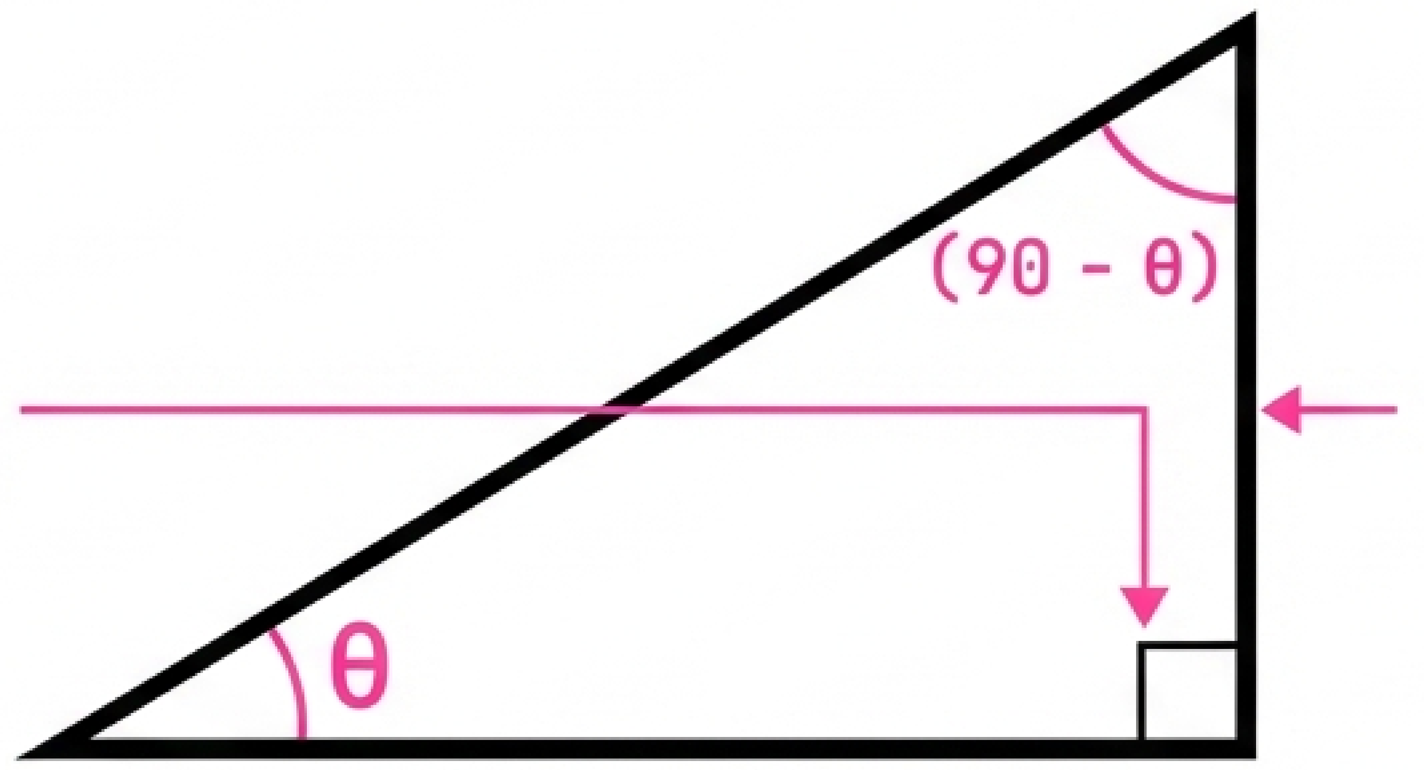
Find tan P: → QR / PR

Find tan Q: → PR / QR

The Complementary Pattern

The acute angles sum to 90°. They are complements.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$



$$\cos(90 - \theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$\sin\theta = \cos(90 - \theta)$ ✓

$\cos\theta = \sin(90 - \theta)$ ✓

$\tan\theta \times \tan(90 - \theta) = 1$ ✓

Interconnected Ratios

$$\tan\theta = \sin\theta / \cos\theta$$

$$= (Opp / Hyp) \div (Adj / Hyp)$$

$$= (Opp / Hyp) \times (Hyp / Adj)$$

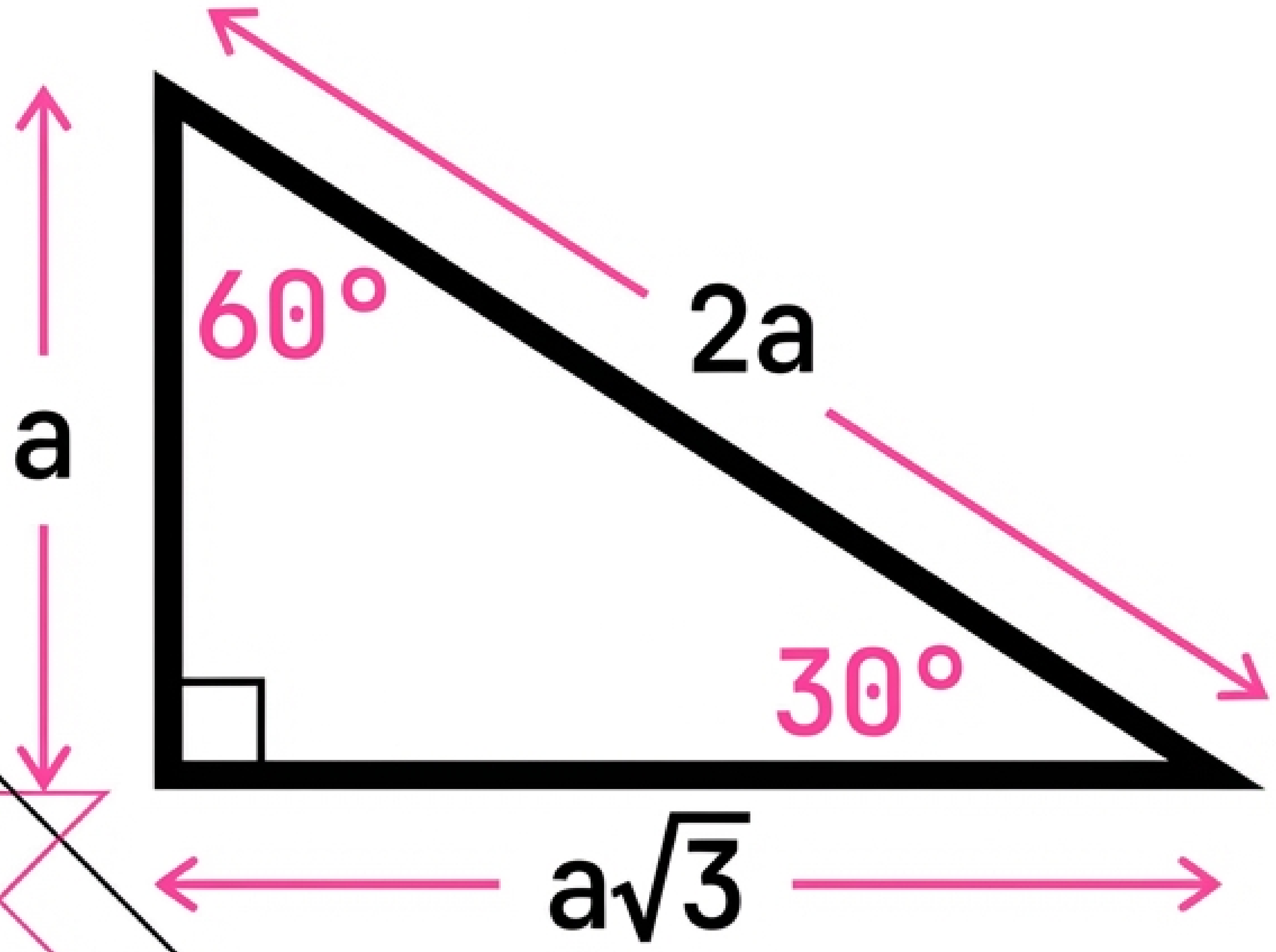
$$= Opp / Adj$$

$$= \tan\theta$$

Inverse Ratios

Cosec = 1/sin
Sec = 1/cos
Cot = 1/tan

Special Angles: 30° and 60°



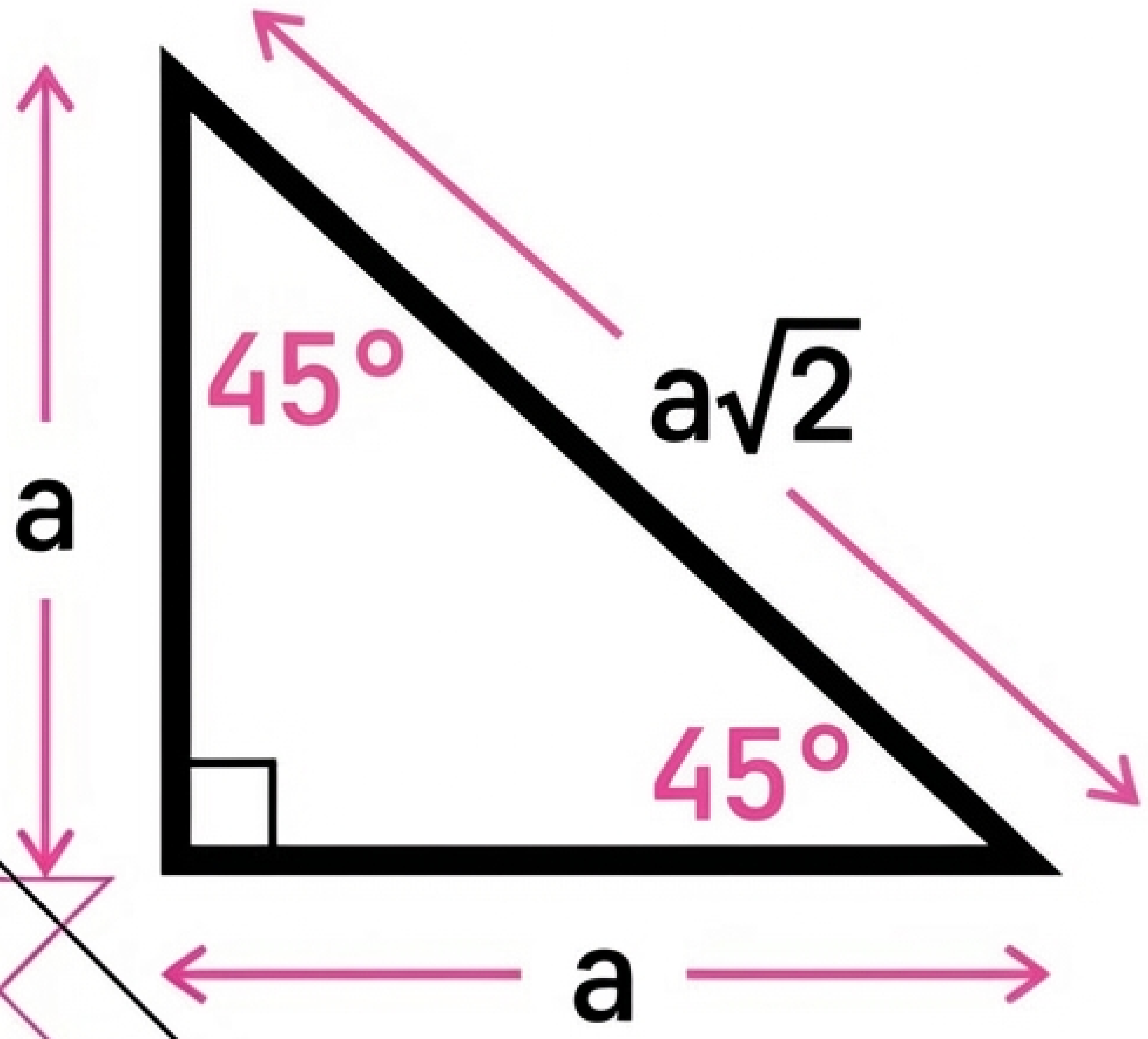
$\sin 30° = 1/2$

$\sin 60° = \sqrt{3}/2$

$\tan 30° = 1/\sqrt{3}$

$\tan 60° = \sqrt{3}$

Special Angles: 45°



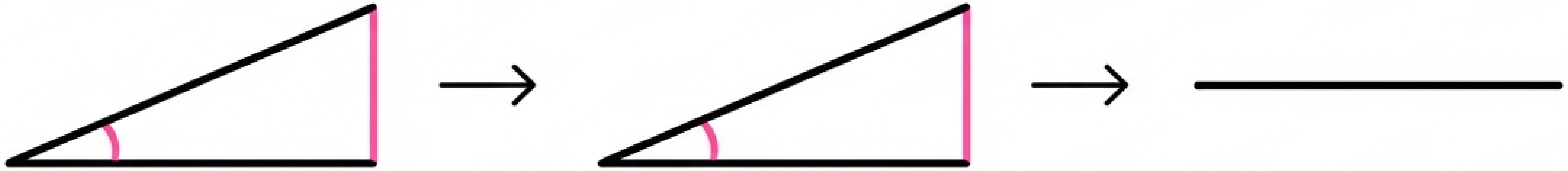
$\sin 45° = 1/\sqrt{2}$

$\cos 45° = 1/\sqrt{2}$

$\tan 45° = 1$

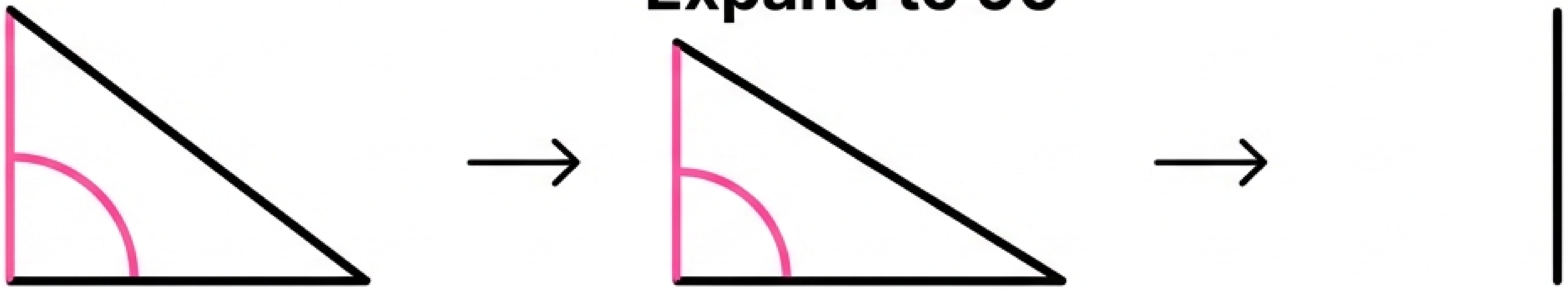
The Extremes: 0° and 90°

Collapse to 0°



sin 0° = 0 cos 0° = 1

Expand to 90°



sin 90° = 1 cos 90° = 0

**tan 90° is Undefined (Division by Zero)

The Trigonometric Table

Ratio	0°	30°	45°	60°	90°
sin θ	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos θ	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
tan θ	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Undefined

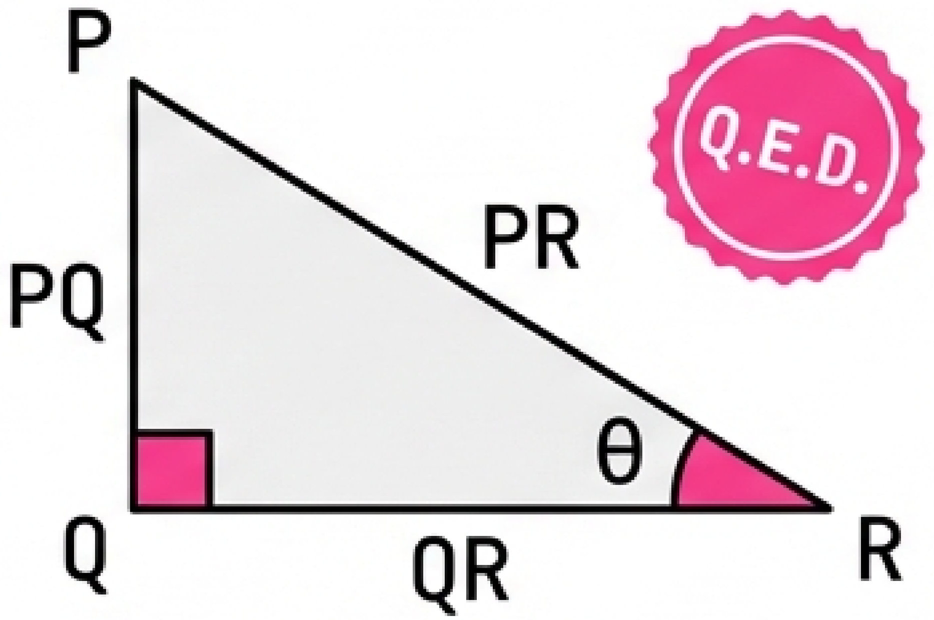
The Fundamental Identity

The most important equation in Trigonometry.

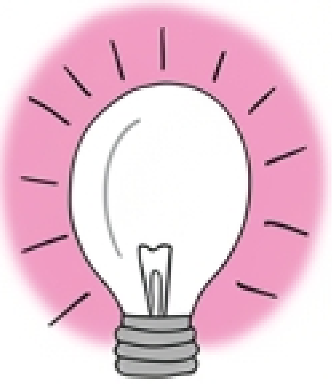
$$\sin^2\theta + \cos^2\theta = 1$$

Visual Proof

1. Pythagoras: $PQ^2 + QR^2 = PR^2$
2. Divide by PR^2: $(PQ/PR)^2 + (QR/PR)^2 = 1$
3. Substitute: $(\sin\theta)^2 + (\cos\theta)^2 = 1$



Worked Example: Connecting the Dots



Problem: If $\sin\theta = \frac{5}{13}$, find $\cos\theta$ and $\tan\theta$.

Step 1: Pythagoras:

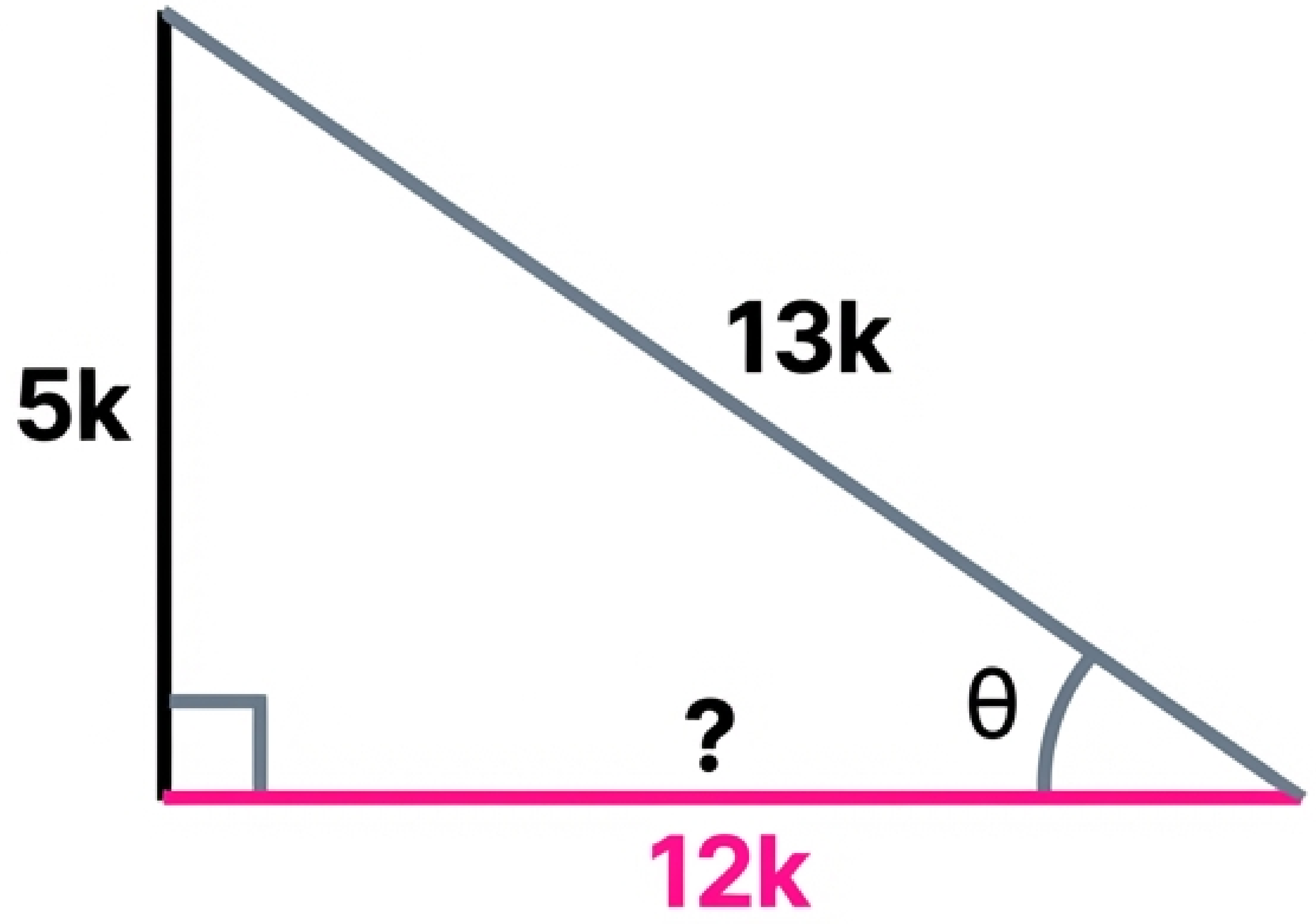
$$\sqrt{(13k)^2 - (5k)^2} = \sqrt{144k^2} = 12k$$

Step 2:

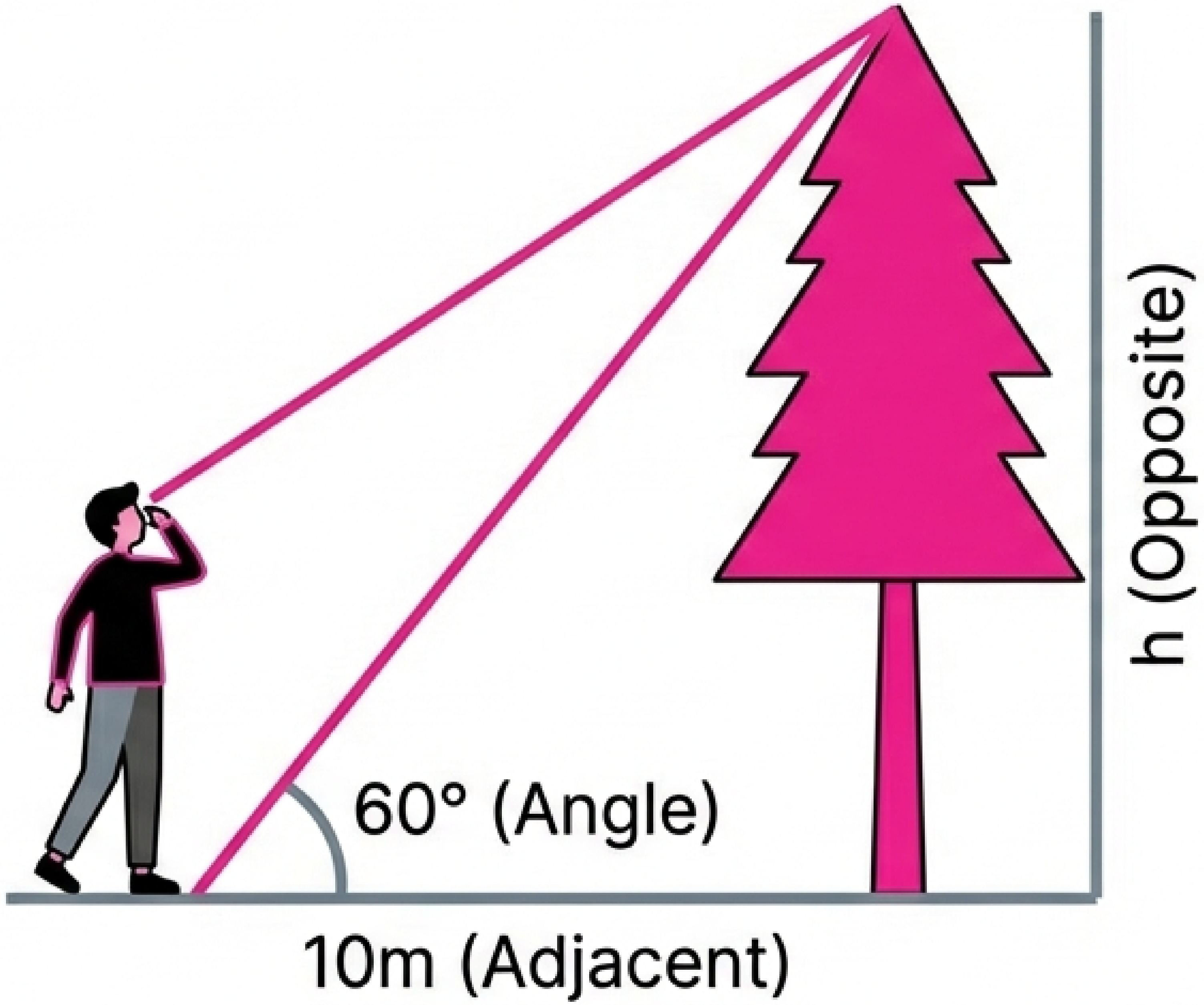
$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} = \frac{12}{13}$$

Step 3:

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} = \frac{5}{12}$$



Application: Solving the Height



- **We know:** Adjacent (10m), Angle (60°).
- **We need:** Opposite (h).
- **Tool:** $\tan\theta = \frac{Opp}{Adj}$

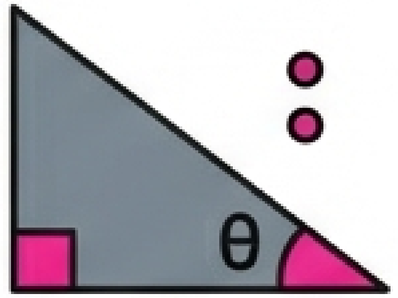
Calculation:

$$\tan 60° = \frac{h}{10}$$

$$\sqrt{3} = \frac{h}{10}$$

$h = 10\sqrt{3}$ meters (approx 17.32m)

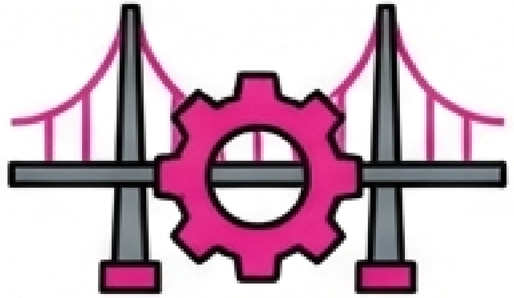
Summary: The Art of Indirect Measurement



Trigonometry links Angles to Side Ratios.

Sine, Cosine, Tangent represent specific relationships.

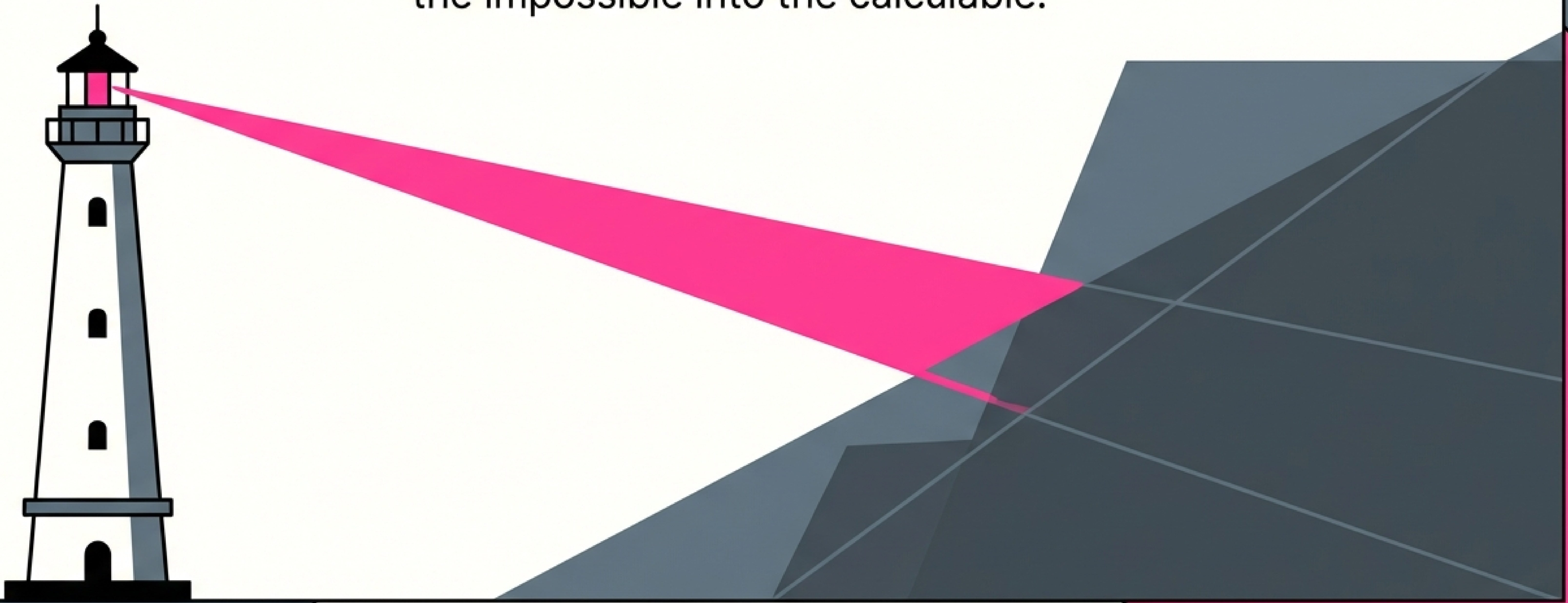
The **Fundamental Identity** connects the ratios.



Used in Engineering, Astronomy, and Navigation.

Seeing the Unseen

Trigonometry allows us to bridge the gap between where we are and where we want to reach. It turns the impossible into the calculable.



[End of Presentation]