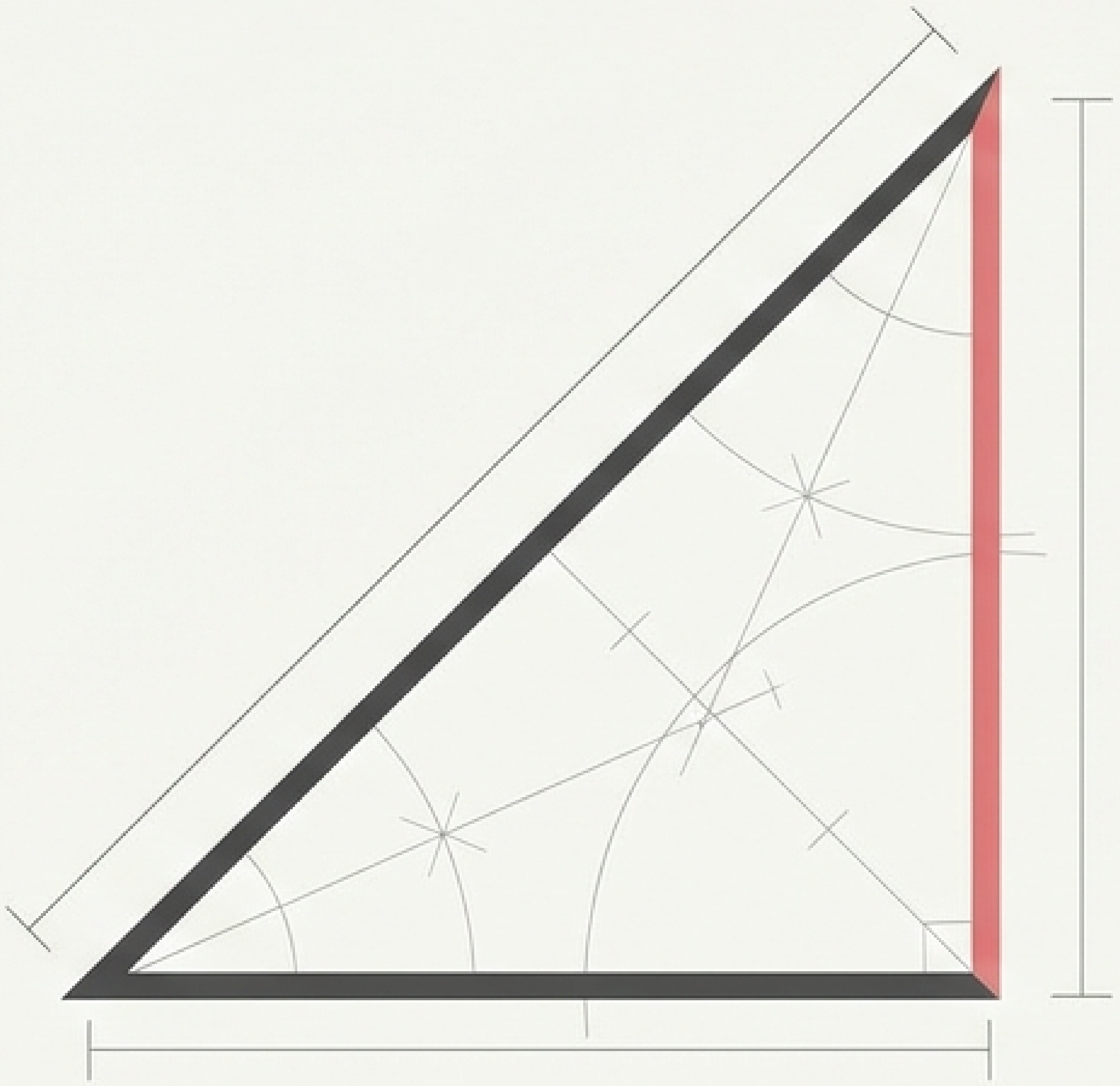


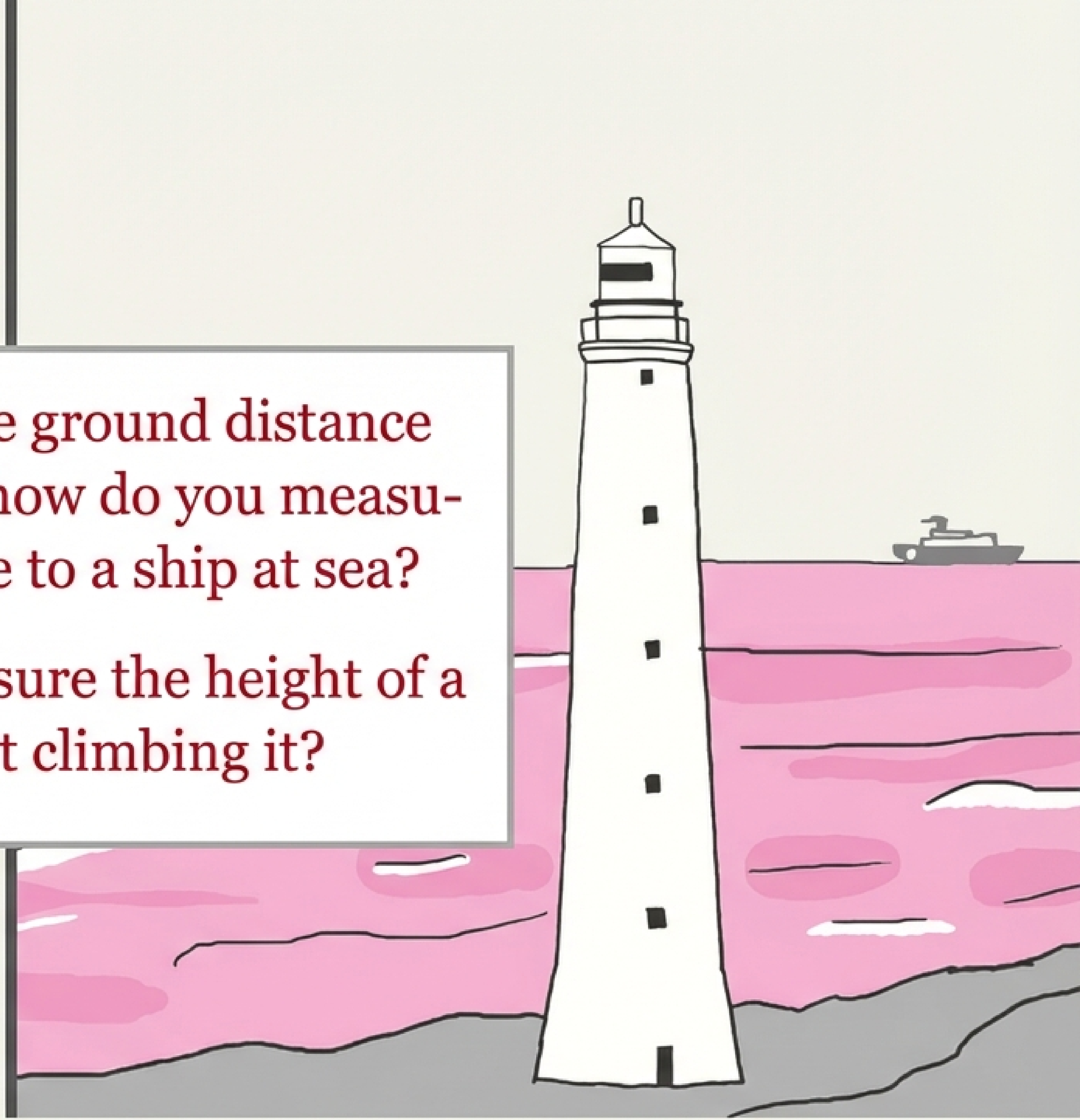
Trigonometry: Measuring the Unmeasurable

A guide to measuring distance without moving.

BASED ON INTRODUCTION TO TRIGONOMETRY, CHAPTER 8

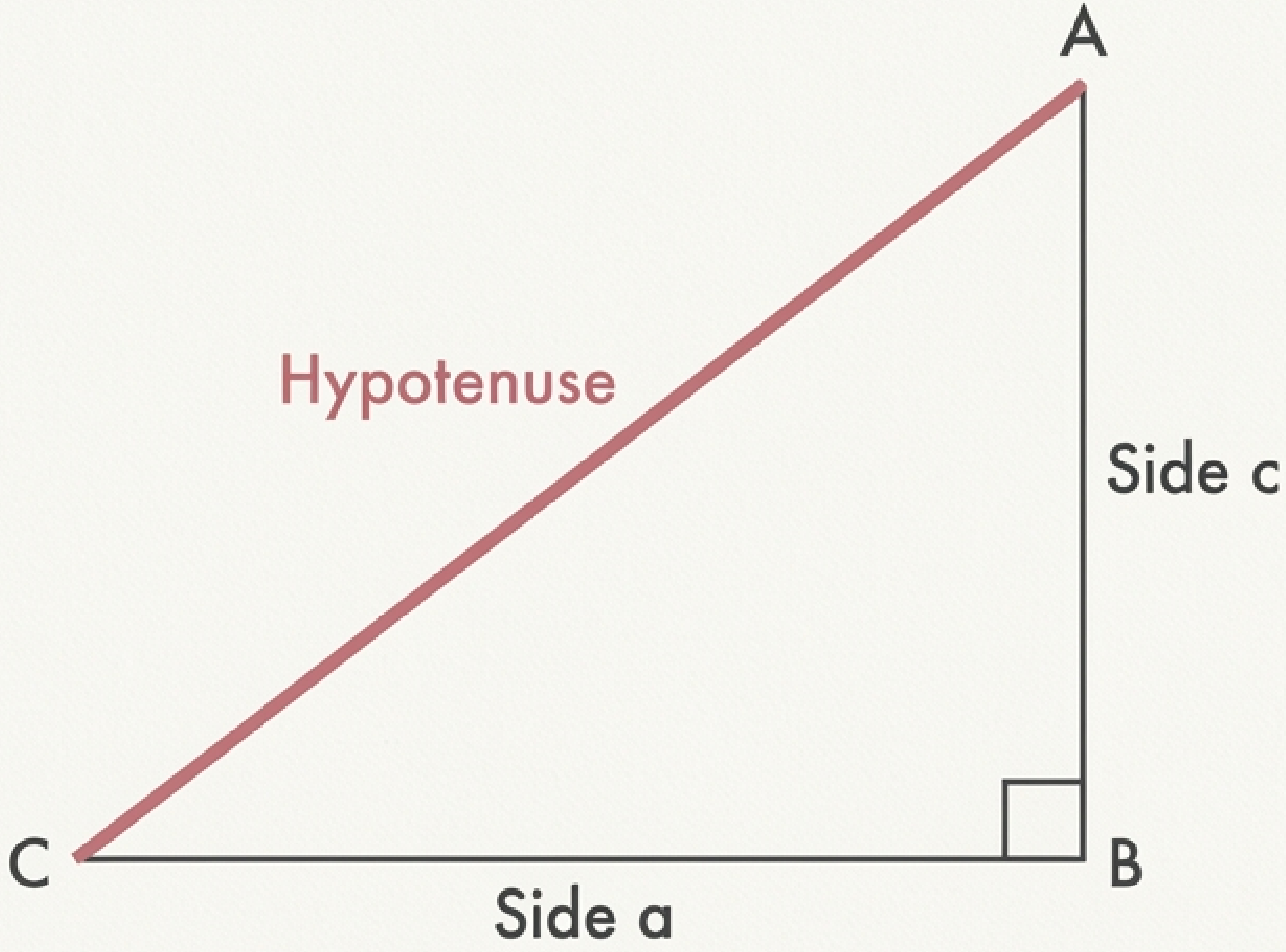
We can measure ground distance with a rope. But how do you measure the distance to a ship at sea?

How do you measure the height of a tree without climbing it?



The answer is Trigonometry. From Greek: Tri (Three) + Gona (Sides) + Metron (Measurements).

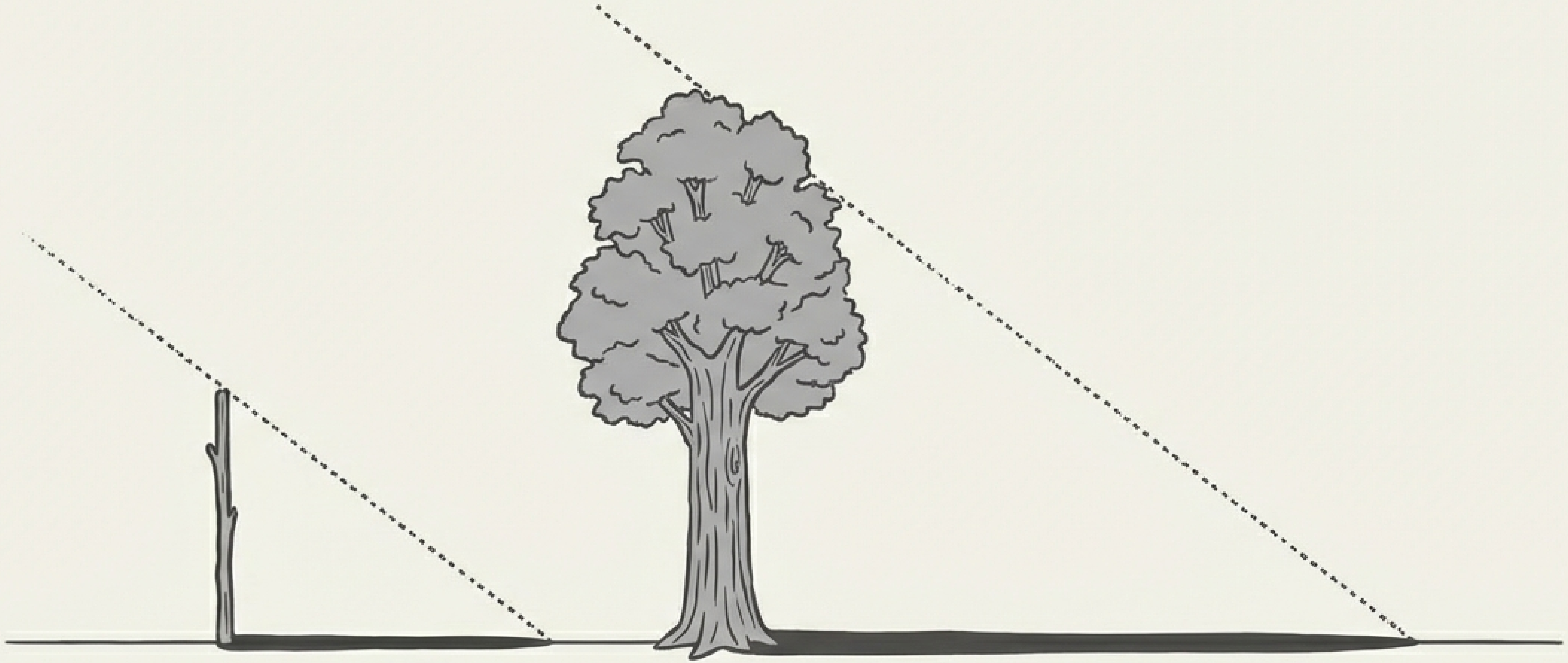
The Foundation: The Right-Angled Triangle



Trigonometry begins here. To measure the unknown, we must first master the relationships within a right-angled triangle.

$$(AB)^2 + (BC)^2 = (AC)^2$$

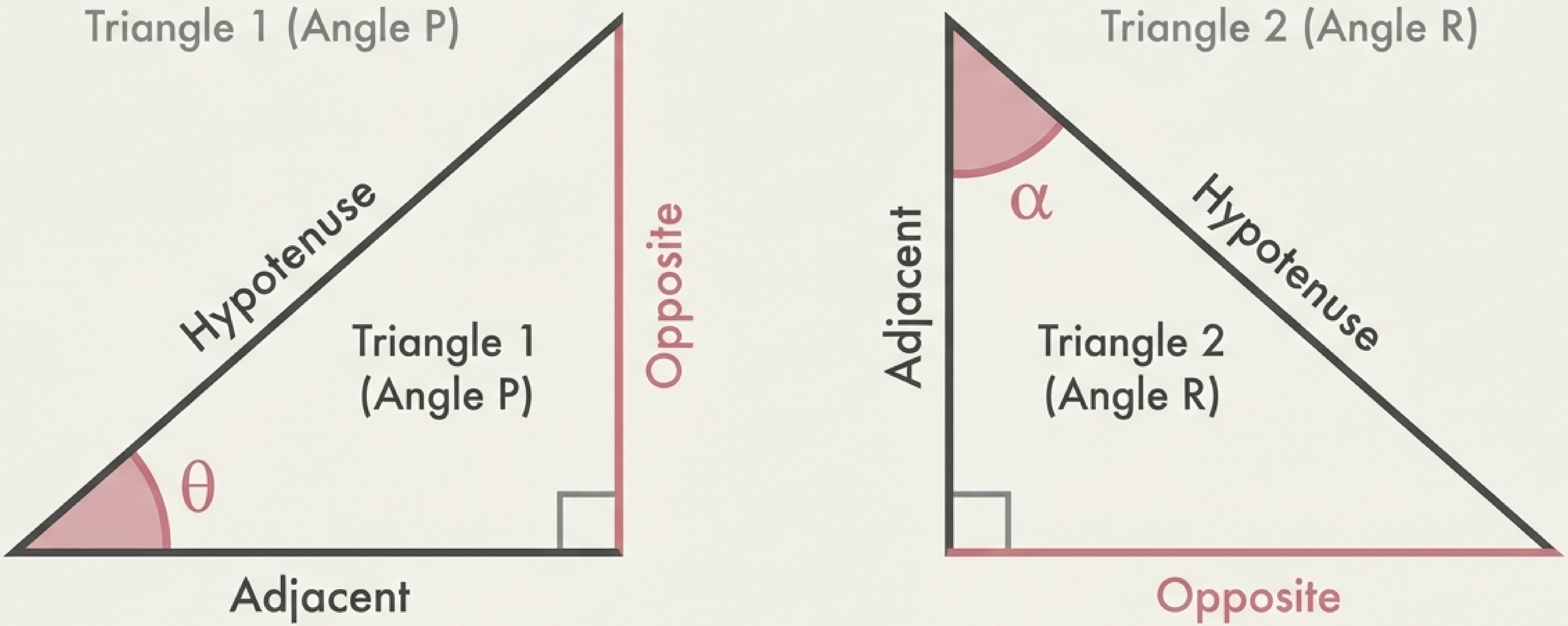
Similar Triangles Unlock the World.



If a stick and a tree intercept parallel sunlight, they form similar triangles (equiangular). Their sides are proportional.

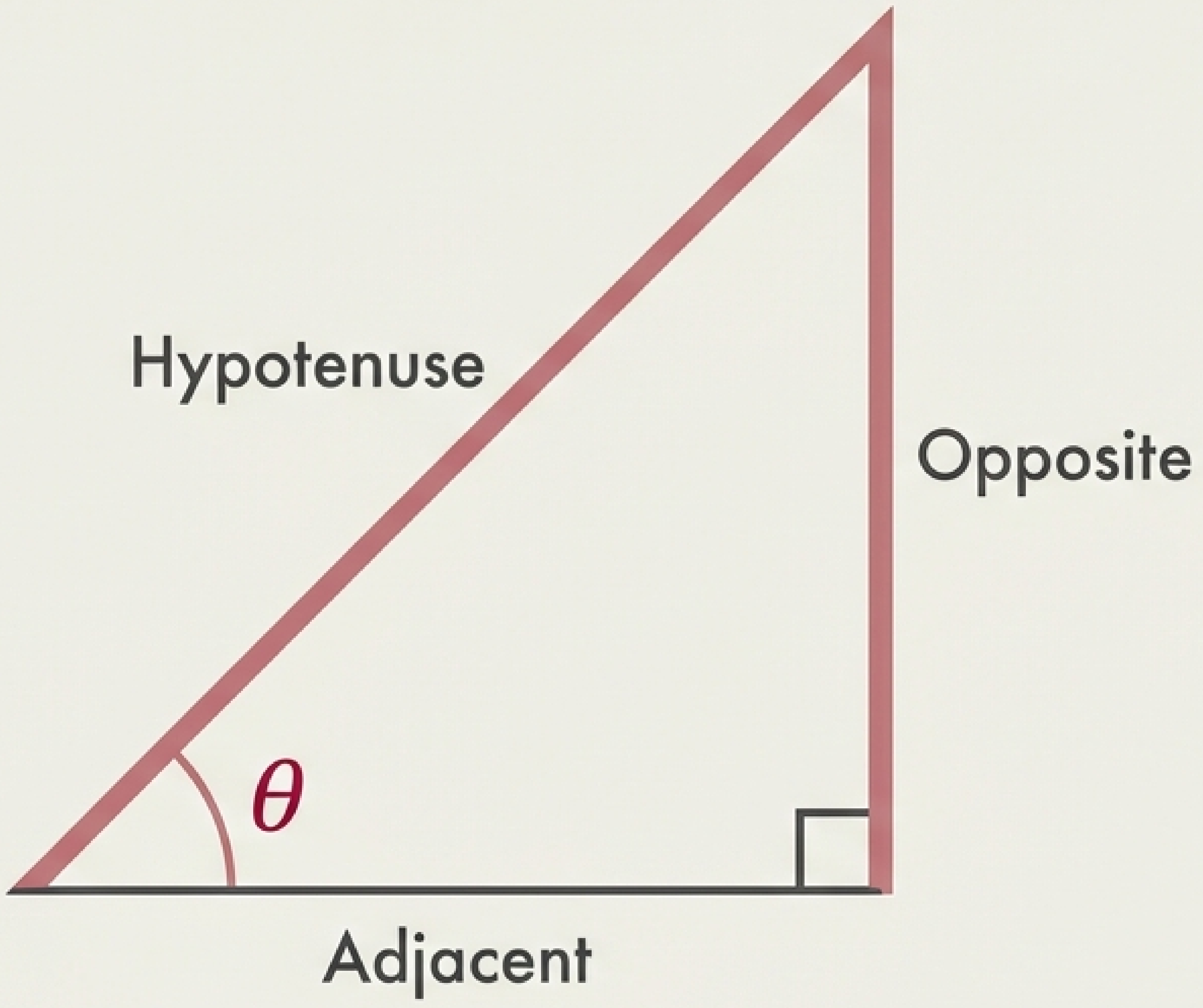
$$\frac{\text{Height of Tree}}{\text{Height of Stick}} = \frac{\text{Shadow of Tree}}{\text{Shadow of Stick}}$$

Anatomy of an Angle



The terms 'Opposite' and 'Adjacent' are relative.
They depend entirely on your perspective.

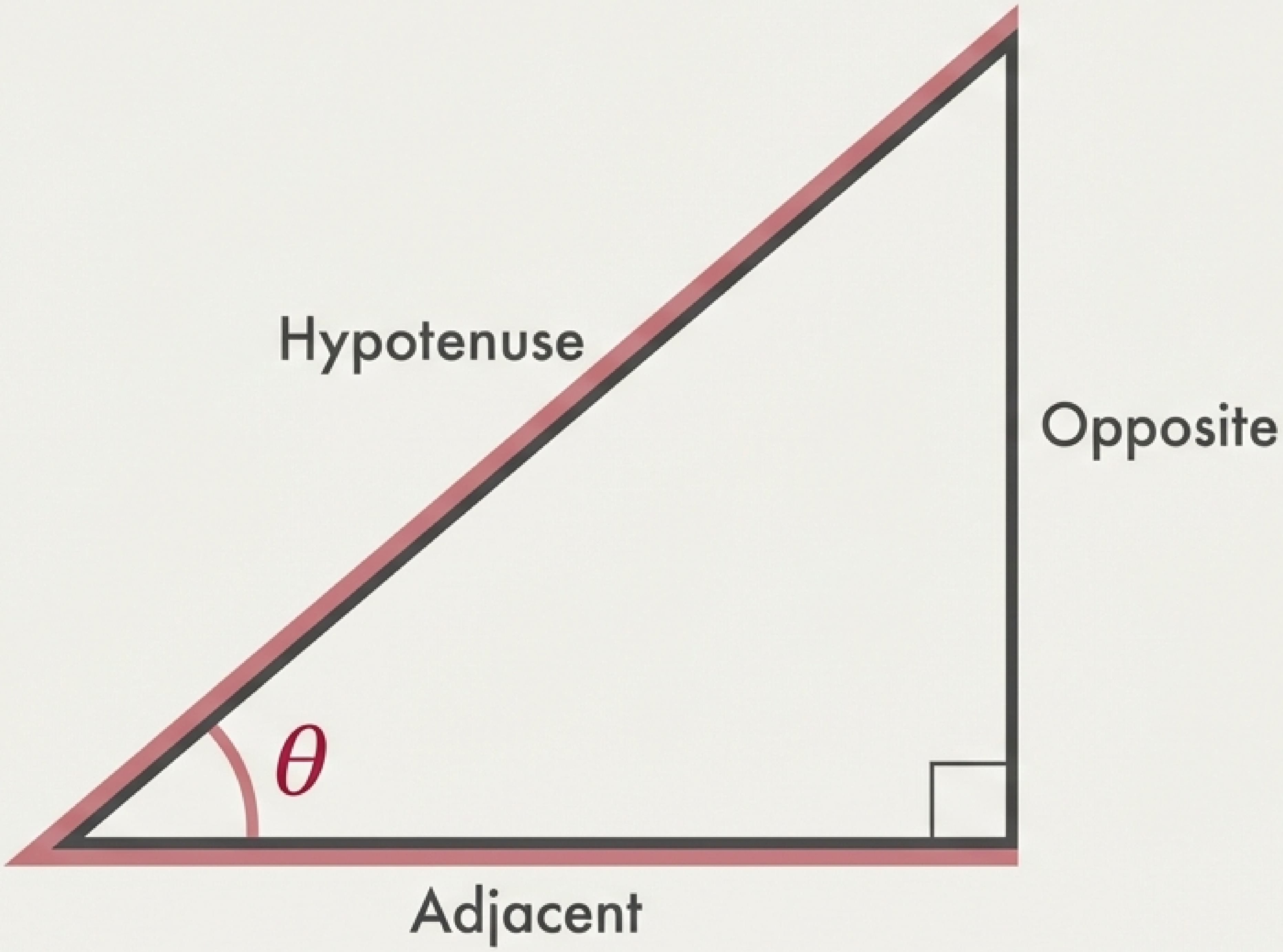
Tool 1: The Sine Ratio



The Sine of angle θ is the ratio of the Opposite side to the Hypotenuse.

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

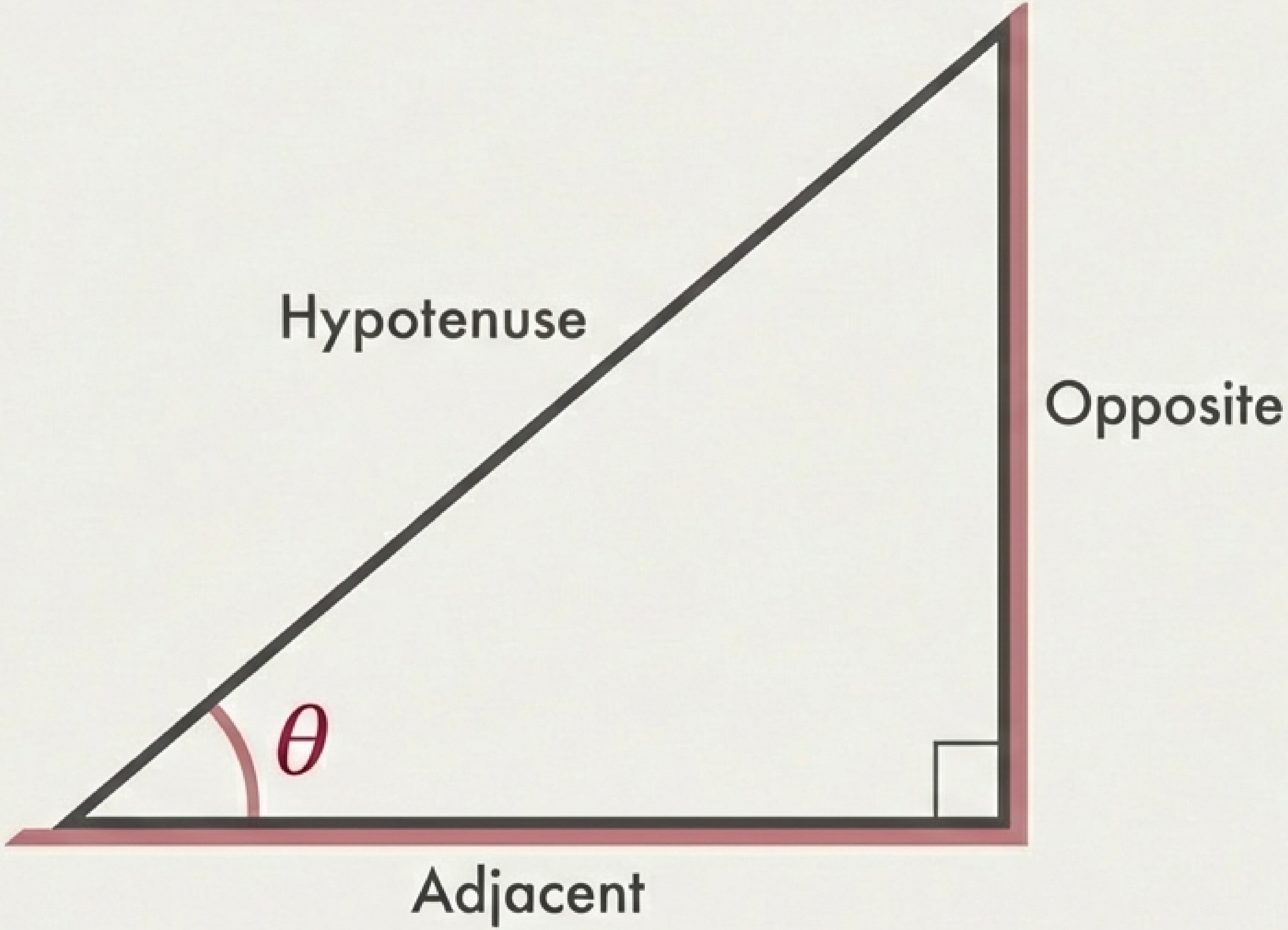
Tool 2: The Cosine Ratio



The Cosine of angle θ is the ratio of the Adjacent side to the Hypotenuse.

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

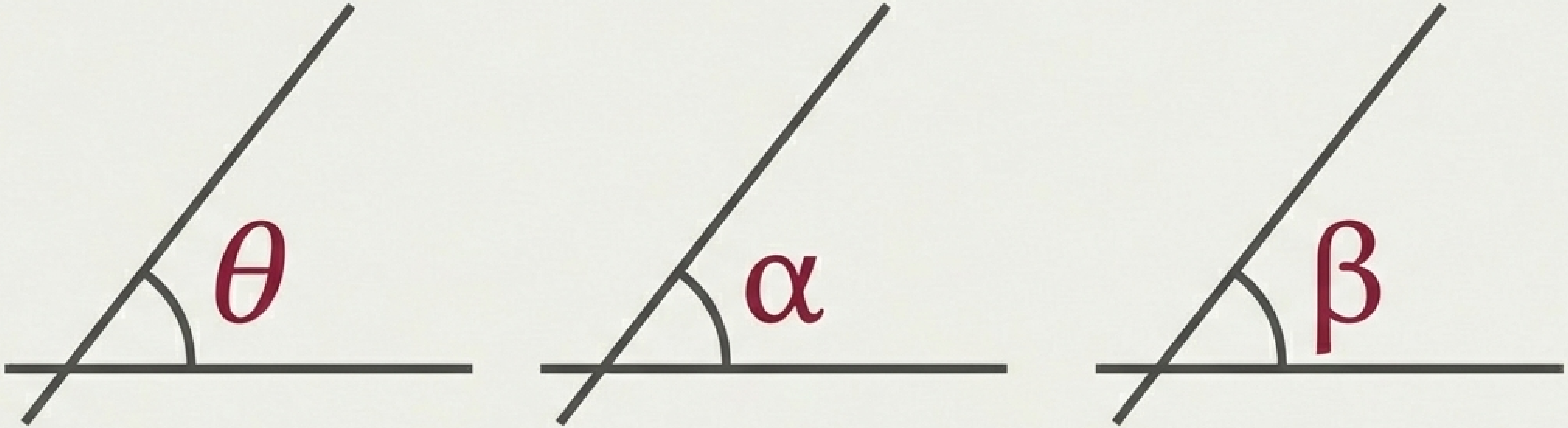
Tool 3: The Tangent Ratio



The Tangent of angle θ is the ratio of the Opposite side to the Adjacent side.

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

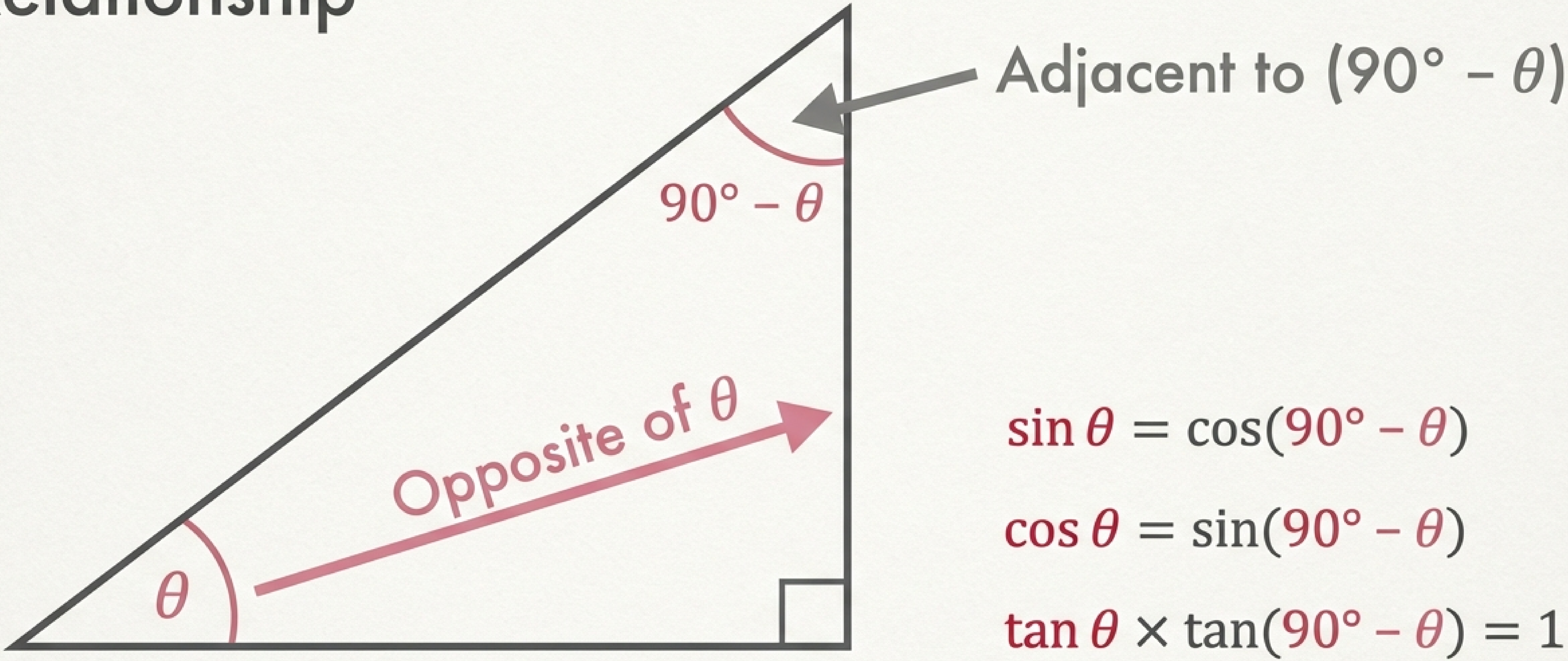
The Language of Angles



In trigonometry, we use Greek letters to denote the measures of acute angles.

$\sin C$ is conventionally written as $\sin\theta$.

The Complementary Relationship Relationship



$$\sin\theta = \cos(90^\circ - \theta)$$

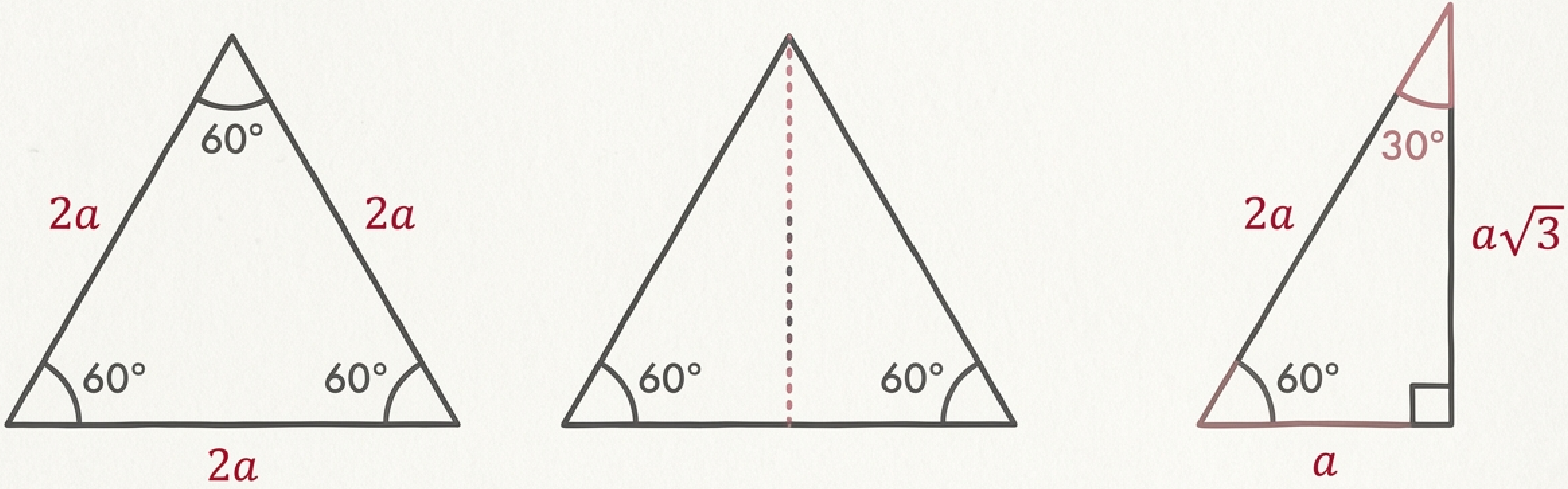
$$\cos\theta = \sin(90^\circ - \theta)$$

$$\tan\theta \times \tan(90^\circ - \theta) = 1$$

Advanced Toolkit: Inverse Ratios

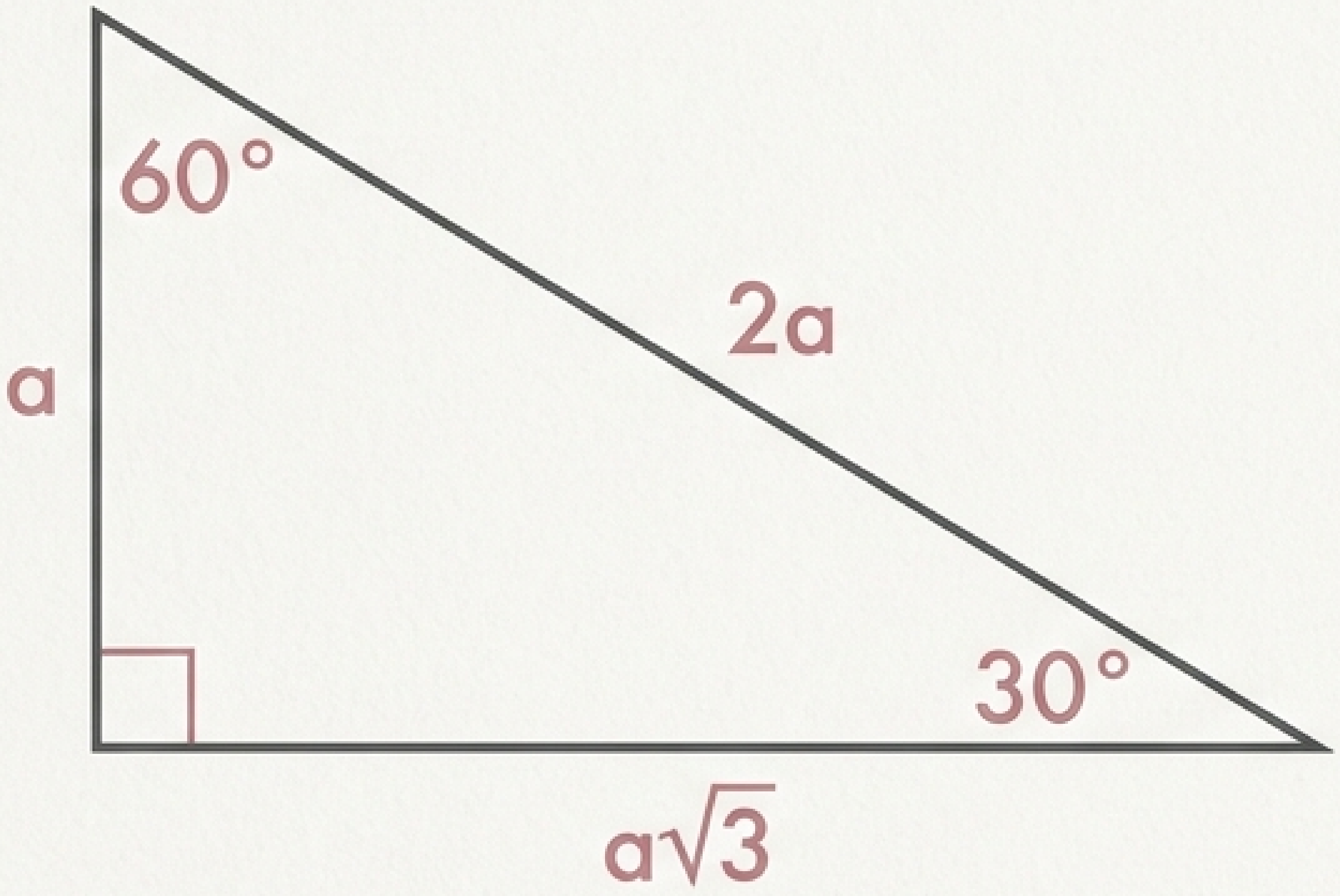
Cosecant (cosec)	Secant (sec)	Cotangent (cot)
The inverse of Sine.	The inverse of Cosine.	The inverse of Tangent.
$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}}$	$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}}$	$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent}}{\text{Opposite}}$

Calibration: Deriving 30° and 60°



By splitting an equilateral triangle, we find the exact dimensions required for 30° and 60° calculations.

The Exact Values for 30° and 60°



30° Values:

$\sin 30° = 1/2$

$\cos 30° = \sqrt{3}/2$

$\tan 30° = 1/\sqrt{3}$

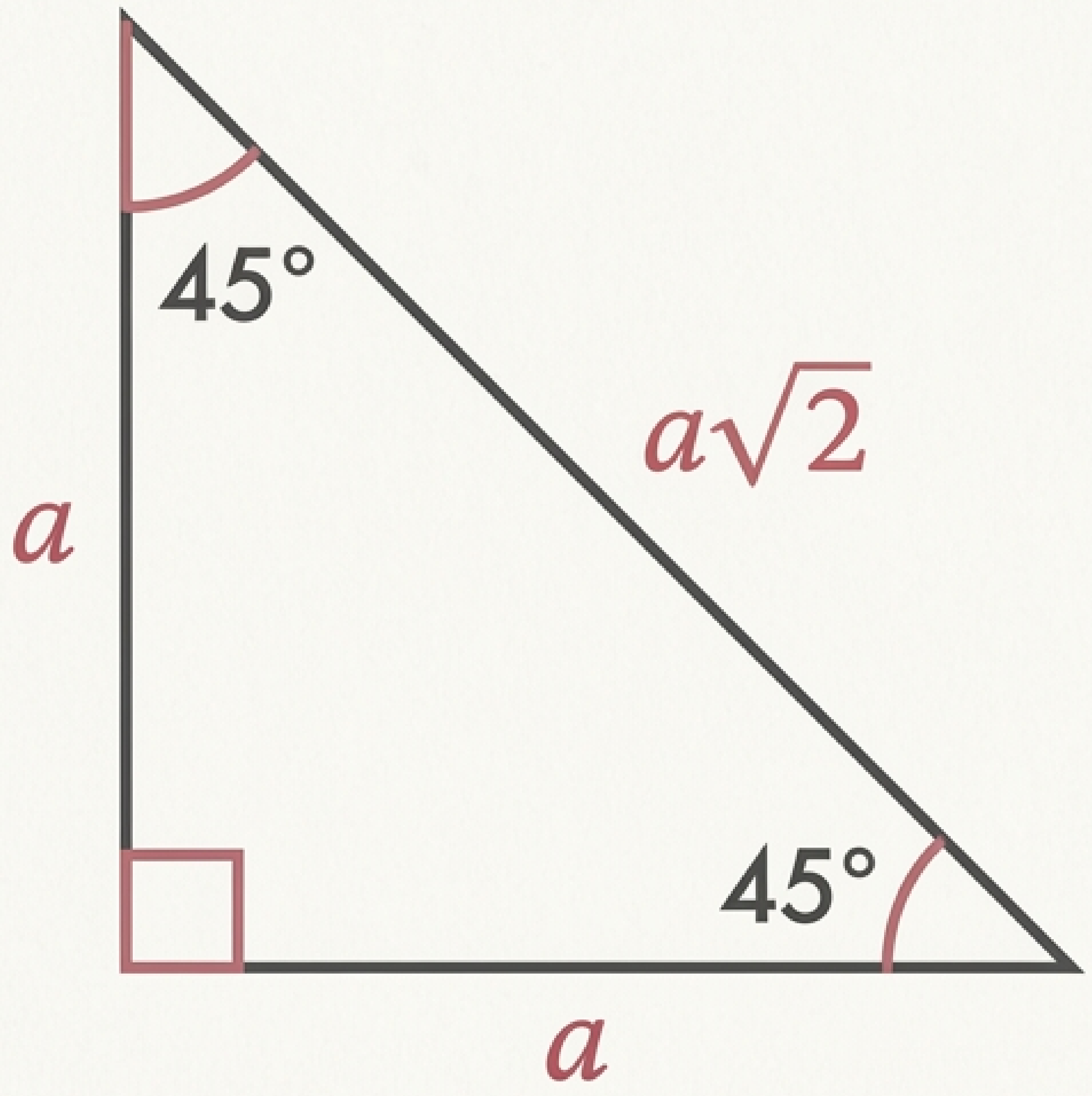
60° Values:

$\sin 60° = \sqrt{3}/2$

$\cos 60° = 1/2$

$\tan 60° = \sqrt{3}$

Calibration: Deriving 45°.



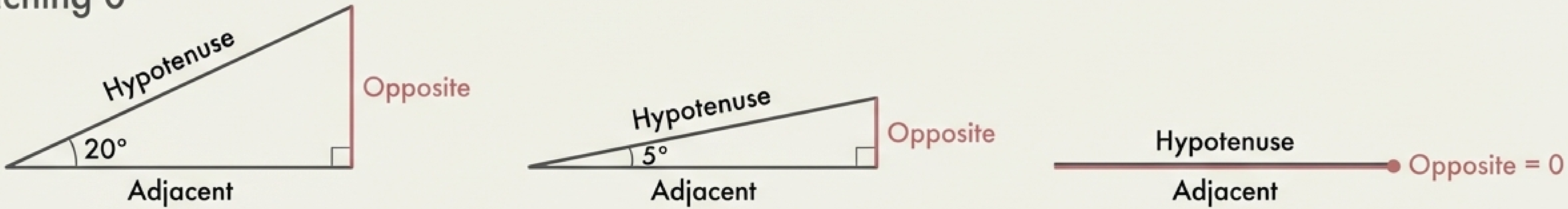
$$\sin 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{a}{a} = 1$$

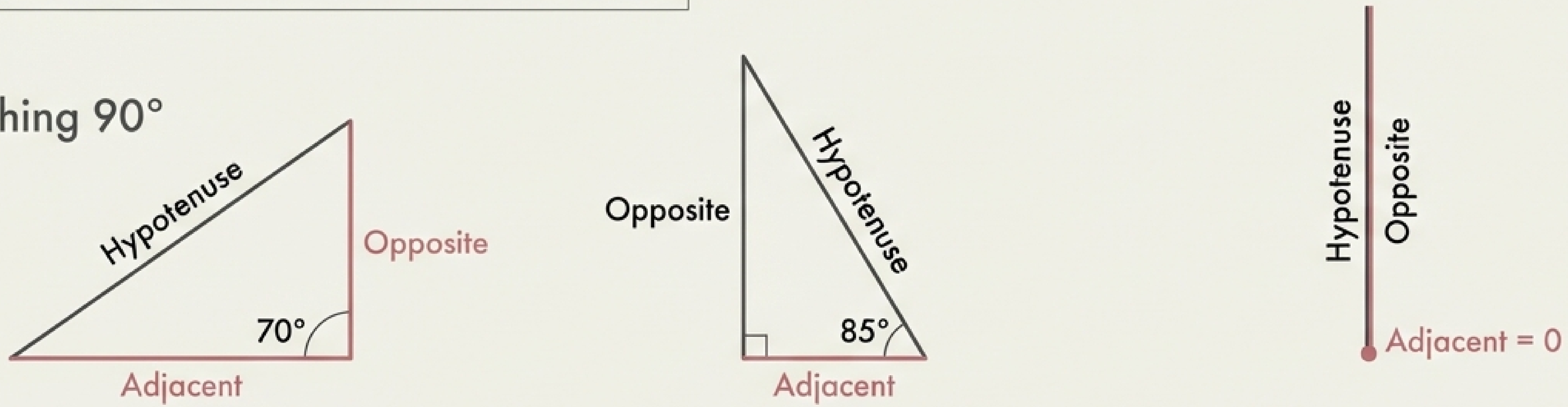
The Limits: 0° and 90°.

Approaching 0°



As angle → 0, Opposite → 0. Therefore $\sin 0 = 0$.

Approaching 90°



As angle → 90, Adjacent → 0. Therefore $\cos 90 = 0$.

$\tan 90°$ is undefined because division by zero (adjacent side) is impossible.

The Trigonometric Table.

Angle θ	0°	30°	45°	60°	90°
sin θ	0	$\sin 30° = \frac{1}{2}$	$\sin 1° = \frac{1}{\sqrt{2}}$	$\sin \theta° = \frac{\sqrt{3}}{2}$	1
cos θ	1	$\cos 3° = \frac{\sqrt{3}}{2}$	$\cos 1° = \frac{1}{\sqrt{2}}$	$\cos 60° = \frac{1}{2}$	0
tan θ	0	$\tan \theta = \frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

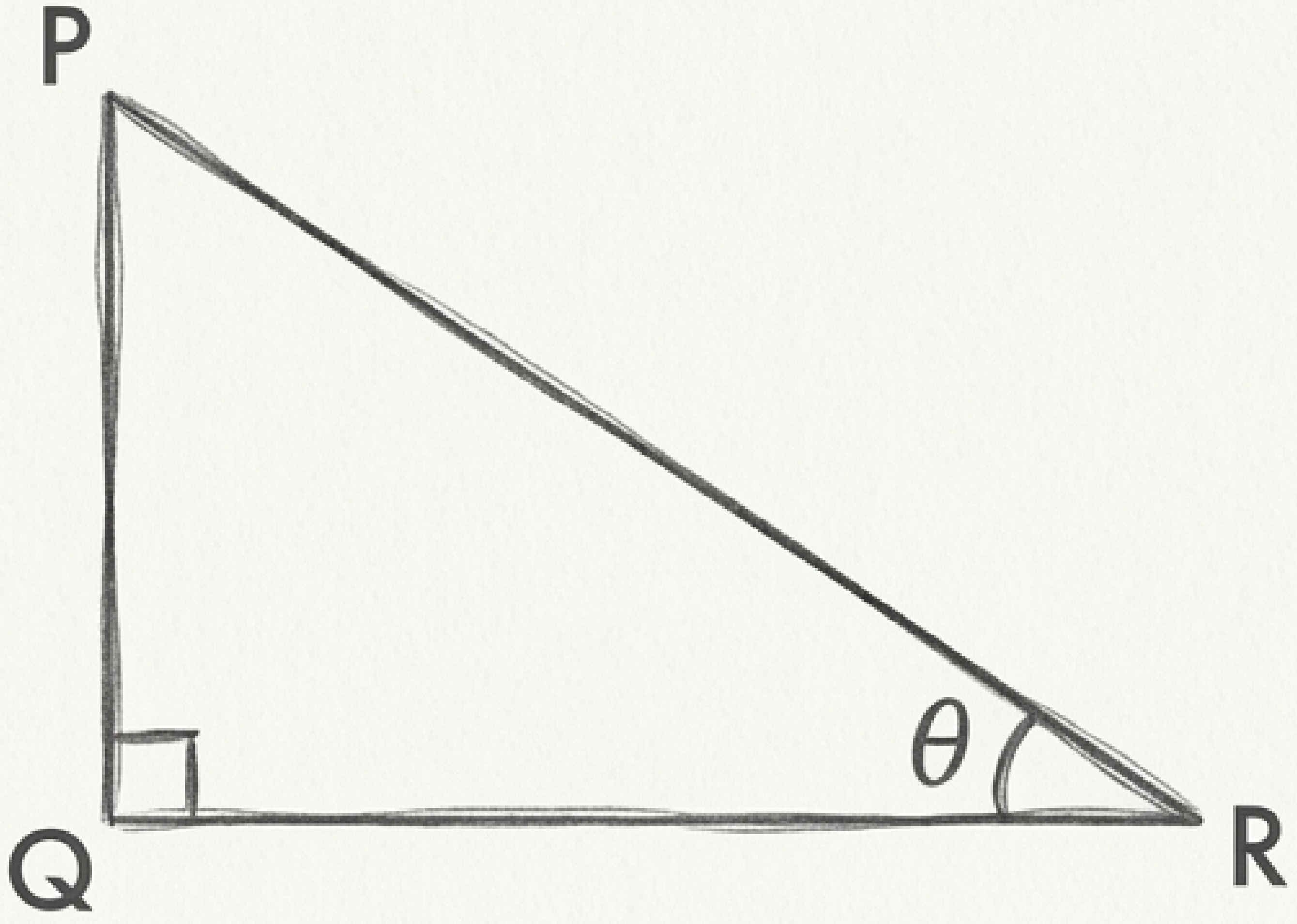
The Fundamental Identity

$$\sin^2\theta + \cos^2\theta = 1$$

For any acute angle θ, the square of the sine plus the square of the cosine always equals unity.

Note: $\sin^2\theta$ is the notation for $(\sin\theta)^2$.

Proving the Identity.



Proof Steps:

1. By Pythagoras:
   $PQ^2 + QR^2 = PR^2$
2. Divide by Hypotenuse squared (PR^2):
   $\frac{PQ^2}{PR^2} + \frac{QR^2}{PR^2} = \frac{PR^2}{PR^2}$
3. Group terms:
   $\left(\frac{PQ}{PR}\right)^2 + \left(\frac{QR}{PR}\right)^2 = 1$
4. Substitute definitions:
   $(\sin\theta)^2 + (\cos\theta)^2 = 1$

Dusty Rose Crimson Pro

Application: Evaluating Expressions.

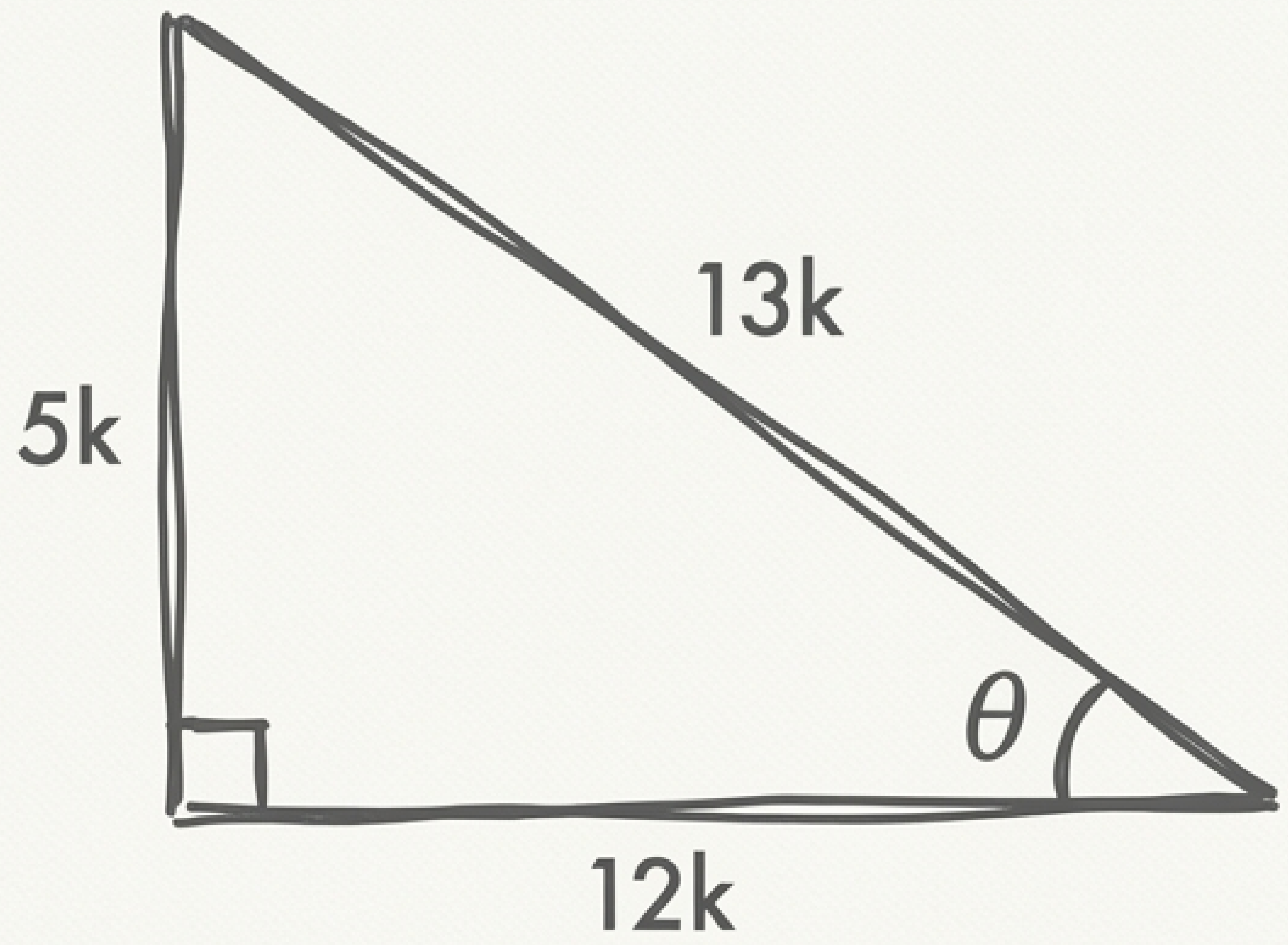
Evaluate: $2\tan 45^\circ + \cos 30^\circ - \sin 60^\circ$

$$= 2(1) + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$= 2 + 0$$

$$= 2$$

Application: Finding Unknown Ratios.



Given: $\sin\theta = 5/13$.
Find $\cos\theta$ and $\tan\theta$.

Step 1: Use Pythagoras to find the adjacent side.

$(13k)^2 - (5k)^2 = 144k^2$

Adjacent = 12k

Step 2: Calculate Ratios.

$\cos\theta = 12k/13k = 12/13$

$\tan\theta = 5k/12k = 5/12$

The Toolkit Summary.

Sine Formula

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\text{O}}{\text{H}}$$

Cosine Formula

$$\cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\text{A}}{\text{H}}$$

Tangent Formula

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{\text{O}}{\text{A}}$$

Pythagorean Identity

$$\sin^2\theta + \cos^2\theta = 1$$

Complementary Rule

$$\sin\theta = \cos(90^\circ - \theta)$$

"From shadows to stars, these ratios allow us to measure the universe."