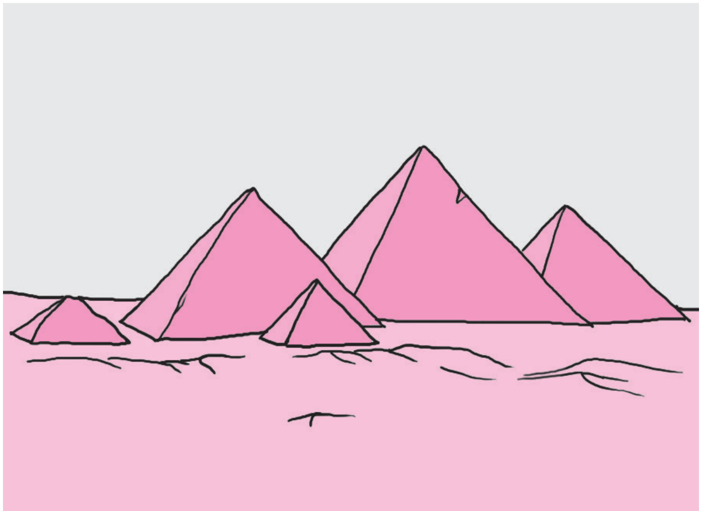
1 Basic Concepts in Geometry

Let's study.

- Point, line and plane
- Co-ordinates of a points and distance
- Betweenness
- Conditional statements
- Proof



Did you recognise the adjacent picture ? It is a picture of pyramids in Egypt, built 3000 years before Christian Era. How the people were able to build such huge structures in so old time ? It is not possible to build such huge structures without developed knowledge of Geometry and Engineering.

The word Geometry itself suggests the origin of the subject. It is generated from the Greek words Geo (Earth) and Metria (measuring). So it can be guessed that the subject must have evolved from the need of measuring the Earth, that is land .

Geometry was developed in many nations in different periods and for different constructions. The first Greek mathematician, Thales, had gone to Egypt. It is said that he determined height of a pyramid by measuring its shadow and using properties of similar triangles.

Ancient Indians also had deep knowledge of Geometry. In vedic period, people used geometrical properties to build altars. The book shulba-sutra describes how to build different shapes by taking measurements with the help of a string. In course of time, the mathematicians Aaryabhat, Varahamihir, Bramhagupta, Bhaskaracharya and many others have given valuable contribution to the subject of Geometry.

Let's learn.

Basic concepts in geometry (Point, Line and Plane)

We do not define numbers. Similarly we do not define a point, line and plane also. These are some basic concepts in Geometry. Lines and planes are sets of points. Keep in mind that the word 'line' is used in the sense 'straight line'.

Co-ordinates of points and distance

Observe the following number line.

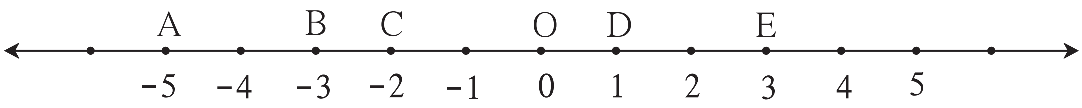


Fig. 1.1

Here, the point D on the number line denotes the number 1. So, it is said that 1 is the co-ordinate of point D. The point B denotes the number – 3 on the line. Hence the co-ordinate of point B is – 3. Similarly the co-ordinates of point A and E are – 5 and 3 respectively.

The point E is 2 unit away from point D. It means the distance between points D and E is 2. Thus, we can find the distance between two points on a number line by counting number of units. The distance between points A and B on the above number line is also 2.

Now let us see how to find distance with the help of co-ordinates of points.

To find the distance between two points, consider their co-ordinates and subtract the smaller co-ordinate from the larger.

The co-ordinates of points D and E are 1 and 3 respectively. We know that 3 > 1.

Therefore, distance between points E and D = 3 – 1 = 2

The distance between points E and D is denoted as d (E,D). This is the same as l(ED), that is, the length of the segment ED.

d (E, D) = 3 – 1 = 2

∴ l(ED) = 2

d (E, D) = l(ED) = 2

Similarly d (D, E) = 2

d (C, D) = 1 – (– 2)

= 1 + 2 = 3

∴ d (C, D) = l(CD) = 3

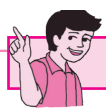
Similarly d (D, C) = 3

Now, let us find d(A,B). The co-ordinate of A is – 5 and that of B is – 3; – 3 > – 5

∴ d (A, B) = – 3 – (– 5) = – 3+5 = 2.

From the above examples it is clear that the distance between two distinct points is always a positive number.

Note that, if the two points are not distinct then the distance between them is zero.



Remember this !

- The distance between two points is obtained by subtracting the smaller co-ordiante from the larger co-ordinate.
- The distance between any two points is a non-negative real number.

Let's learn.

Betweenness

If P, Q, R are three distinct collinear points, there are three possibilities.

Fig. 1.2

(i) Point Q is between P and R

(ii) Point R is between P and Q

(iii) Point P is between R and Q

If d (P, Q) + d (Q, R) = d (P, R) then it is said that point Q is between P and R. The betweeness is shown as P - Q - R.

Solved examples

Ex (1) On a number line, points A, B and C are such that

d (A, B) = 5, d (B,C) = 11 and d (A, C) = 6.

Which of the points is between the other two ?

Solution : Which of the points A, B and C is between the other two, can be decided as follows.

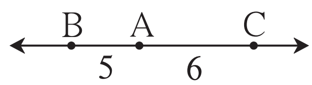


Fig. 1.3

d(B,C) = 11 . . . . (I)

d(A,B) + d(A,C)= 5+6 = 11 . . . . (II)

$\therefore$ d (B, C) = d (A, B) + d (A, C) . . . . [from (I) and (II)]

Point A is between point B and point C.

Ex (2) U, V and A are three cities on a straight road. The distance between U and A is 215 km, between V and A is 140 km and between U and V is 75 km. Which of them is between the other two ?

Solution : d (U,A) = 215; d (V,A) = 140; d (U,V) = 75

d (U,V) + d (V,A) = 75 + 140 = 215; d (U,A) = 215

$\therefore$ d (U,A) = d (U,V) + d (V,A)

$\therefore$ The city V is between the cities U and A.

Ex (3) The co-ordinate of point A on a number line is 5. Find the co-ordinates of points on the same number line which are 13 units away from A.

Solution : As shown in the figure, let us take points T and D to the left and right of A respectively, at a distance of 13 units.

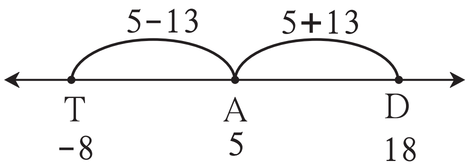


Fig. 1.4

The co-ordinate of point T, which is to the left of A ,will be $5 - 13 = -8$

The co-ordinate of point D, which is to the right of A, will be $5 + 13 = 18$

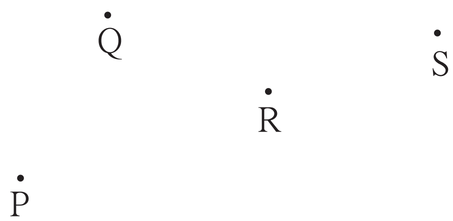
$\therefore$ the co-ordinates of points 13 units away from A will be -8 and 18.

Verify your answer : $d(A,D) = d(A,T) = 13$

Activity

(1) Points A, B, C are given aside. Check, with a stretched thread, whether the three points are collinear or not. If they are collinear, write which one of them is between the other two.

(2) Given aside are four points P, Q, R, and S. Check which three of them are collinear and which three are non collinear. In the case of three collinear points, state which of them is between the other two.



(3) Students are asked to stand in a line for mass drill. How will you check whether the students standing are in a line or not ?

(4) How had you verified that light rays travel in a straight line ?

Recall an experiment in science which you have done in a previous standard.

Practice set 1.1

1. Find the distances with the help of the number line given below.

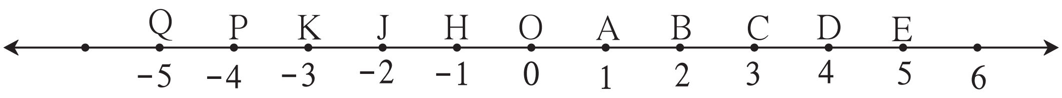


Fig. 1.5

(i) d(B,E) (ii) d(J, A) (iii) d(P, C) (iv) d(J, H)

(v) d(K, O) (vi) d(O, E) (vii) d(P, J) (viii) d(Q, B)

2. If the co-ordinate of A is x and that of B is y, find d(A, B) .

(i) $x = 1, y = 7$ (ii) $x = 6, y = -2$ (iii) $x = -3, y = 7$

(iv) $x = -4, y = -5$ (v) $x = -3, y = -6$ (vi) $x = 4, y = -8$

3. From the information given below, find which of the point is between the other two. If the points are not collinear, state so.

(i) $d(P, R) = 7$, $d(P, Q) = 10$, $d(Q, R) = 3$

(ii) $d(R, S) = 8$, $d(S, T) = 6$, $d(R, T) = 4$

(iii) $d(A, B) = 16$, $d(C, A) = 9$, $d(B, C) = 7$

(iv) $d(L, M) = 11$, $d(M, N) = 12$, $d(N, L) = 8$

(v) $d(X, Y) = 15$, $d(Y, Z) = 7$, $d(X, Z) = 8$

(vi) $d(D, E) = 5$, $d(E, F) = 8$, $d(D, F) = 6$

4. On a number line, points A, B and C are such that $d(A,C) = 10$, $d(C,B) = 8$ Find d(A, B) considering all possibilities.

5. Points X, Y, Z are collinear such that $d(X,Y) = 17$, $d(Y,Z) = 8$, find d(X,Z) .

6. Sketch proper figure and write the answers of the following questions.

(i) If A - B - C and $l(AC) = 11$, $l(BC) = 6.5$, then $l(AB)$ =?

(ii) If R - S - T and $l(ST) = 3.7$, $l(RS) = 2.5$, then $l(RT)$ =?

(iii) If X - Y - Z and $l(XZ) = 3\sqrt{7}$, $l(XY) = \sqrt{7}$, then $l(YZ)$ =?

7. Which figure is formed by three non-collinear points ?

Let's learn.

In the book, Mathematics - Part I for std IX, we have learnt union and intersection of sets in the topic on sets. Now, let us describe a segment, a ray and a line as sets of points.

(1) Line segment :

The union set of point A, point B and points between A and B is called segment AB. Segment AB is written as seg AB in brief. Seg AB means seg BA. Point A and point B are called the end points of seg AB. The distance between the end points of a segment is called the length of the segment. That is $l(AB) = d(A,B)$

$l(AB) = 5$ is also written as AB = 5.

Fig. 1.6

(2) Ray AB :

Suppose, A and B are two distinct points. The union set of all points on seg AB and the points P such that A - B - P, is called ray AB. Here point A is called the starting point of ray AB.

Fig. 1.7

(3) Line AB :

The union set of points on ray AB and opposite ray of ray AB is called line AB. The set of points of seg AB is a subset of points of line AB.

(4) Congruent segments :

If the length of two segments is equal then the two segments are congruent.

If $l(AB) = l(CD)$ then seg AB $\cong$ seg CD

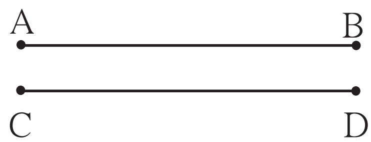


Fig. 1.8

(5) Properties of congruent segements :

(i) Reflexivity : seg AB $\cong$ seg AB

(ii) Symmetry : If seg AB $\cong$ seg CD then seg CD $\cong$ seg AB

(iii) Transitivity : If seg AB $\cong$ seg CD and seg CD $\cong$ seg EF then seg AB $\cong$ seg EF

(6) Midpoint of a segment :

If A-M-B and seg AM $\cong$ seg MB, then M is called the midpoint of seg AB.

Every segment has one and only one midpoint.

Fig. 1.9

(7) Comparison of segments :

If length of segment AB is less than the length of segment CD, it is written as seg AB < seg CD or seg CD > seg AB.

The comparison of segments depends upon their lengths.

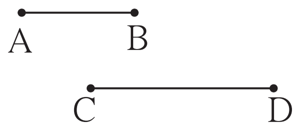


Fig. 1.10

(8) Perpendicularity of segments or rays :

If the lines containing two segments, two rays or a ray and a segment are perpendicular to each other then the two segments, two rays or the segment and the ray are said to be perpendicular to each other.

In the figure 1.11, seg AB $\perp$ line CD,

seg AB $\perp$ ray CD.

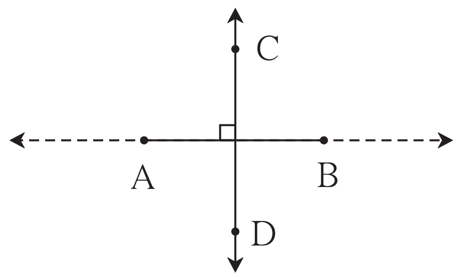


Fig. 1.11

(9) Distance of a point from a line :

If seg CD $\perp$ line AB and the point D lies on line AB then the length of seg CD is called the distance of point C from line AB.

The point D is called the foot of the perpendicular.

If l(CD) = a, then the point C is at a distance of 'a' from the line AB.

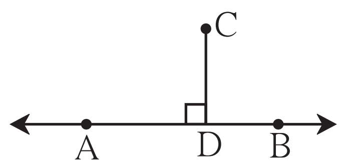


Fig. 1.12

Practice set 1.2

1. The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.

Point	A	B	C	D	E
Co-ordinate	–3	5	2	–7	9

(i) seg DE and seg AB (ii) seg BC and seg AD (iii) seg BE and seg AD

2. Point M is the midpoint of seg AB. If AB = 8 then find the length of AM.
3. Point P is the midpoint of seg CD. If CP = 2.5, find l(CD).
4. If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments.

5. Write the answers to the following questions with reference to figure 1.13.

(i) Write the name of the opposite ray of ray RP

(ii) Write the intersection set of ray PQ and ray RP.

(iii) Write the union set of seg PQ and seg QR.

(iv) State the rays of which seg QR is a subset.

(v) Write the pair of opposite rays with common end point R.

(vi) Write any two rays with common end point S.

(vii) Write the intersection set of ray SP and ray ST.

Fig. 1.13

6. Answer the questions with the help of figure 1.14.

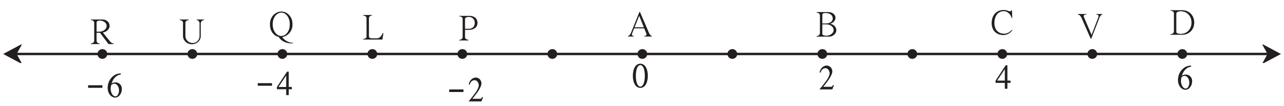
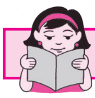


Fig. 1.14

(i) State the points which are equidistant from point B.

(ii) Write a pair of points equidistant from point Q.

(iii) Find *d* (U,V), *d* (P,C), *d* (V,B), *d* (U, L).



Let's learn.

Conditional statements and converse

The statements which can be written in the 'If-then' form are called conditional statements. The part of the statement following 'If' is called the antecedent, and the part following 'then' is called the consequent.

For example, consider the statement : The diagonals of a rhombus are perpendicular bisectors of each other.

The statement can be written in the conditional form as, 'If the given quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.'

If the antecedent and consequent in a given conditional statement are interchanged, the resulting statement is called the **converse** of the given statement.

If a conditional statement is true, its converse is not necessarily true. Study the following examples.

Conditional statement : If a quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.

Converse : If the diagonals of a quadrilateral are perpendicular bisectors of each other then it is a rhombus.

In the above example, the statement and its converse are true.

Now consider the following example,

Conditional statement : If a number is a prime number then it is even or odd.

Converse : If a number is even or odd then it is a prime number.

In this example, the statement is true, but its converse is false.

Let's learn.

Proofs

We have studied many properties of angles, triangles and quadrilaterals through activities.

In this standard we are going to look at the subject of Geometry with a different point of view, which was originated by the Greek mathematician Euclid, who lived in the third century before Christian Era. He gathered the knowledge of Geometry prevailing at his time and streamlined it. He took for granted some self evident geometrical statements which were accepted by all and called them **Postulates.** He showed that on the basis of the postulates some more properties can be proved logically.

Properties proved logically are called **Theorems.**

Some of Euclid's postulates are given below.

(1) There are infinite lines passing through a point.

(2) There is one and only one line passing through two points.

(3) A circle of given radius can be drawn taking any point as its centre.

(4) All right angles are congruent with each other.

(5) If two interior angles formed on one side of a transversal of two lines add up to less than two right angles then the lines produced in that direction intersect each other.

Euclid

We have verified some of these postulates through activities.

A property is supposed to be true if it can be proved logically. It is then called a **Theorem.** The logical argument made to prove a theorem is called its **proof.**

When we are going to prove that a conditional statement is true, its antecedent is called 'Given part' and the consequent is called 'the part to be proved'.

There are two types of proofs, **Direct** and **Indirect.**

Let us give a direct proof of the property of angles made by two intersecting lines.

Theorem : The opposite angles formed by two intersecting lines are of equal measures.

Given : Line AB and line CD intersect at point O such that A - O - B, C - O - D.

To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

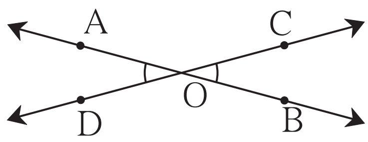


Fig. 1.15

Proof : $\angle AOC + \angle BOC = 180°$ . . . . (I) (angles in linear pair)

$\angle BOC + \angle BOD = 180°$ . . . . (II) (angles in linear pair)

$\angle AOC + \angle BOC = \angle BOC + \angle BOD$ . . . . [from (I) and (II)]

$\therefore \angle AOC = \angle BOD$. . . . . . . eliminating $\angle BOC$.

Similarly, it can be proved that $\angle BOC = \angle AOD$.

Indirect proof :

This type of proof starts with an assumption that the consequence is false. Using it and the properties accepted earlier, we start arguing step by step and reach a conclusion. The conclusion is contradictory with the antecedent or a property which is already accepted. Hence, the assumption that the consequent is false goes wrong. So it is accepted that the consequent is true.

Study the following example.

Statement : A prime number greater than 2 is odd.

Conditional statement : If p is a prime number greater than 2 then it is odd.

Given : p is a prime number greater than 2. That is, 1 and p are the only divisors of p.

To prove : p is an odd number.

Proof : Let us suppose that p is not an odd number.

So p is an even number.

$\therefore$ a divisor of p is 2 ..... (I)

But it is given that p is a prime number greater than 2. ....(given)

$\therefore$ 1 and p are the only divisors of p ..... (II)

Statements (I) and (II) are contradictory.

$\therefore$ the assumption , that p is not odd is false.

This proves that a prime number greater than 2 is odd.

Practice set 1.3

1. Write the following statements in 'if-then' form.

(i) The opposite angles of a parallelogram are congruent.

(ii) The diagonals of a rectangle are congruent.

(iii) In an isosceles triangle, the segment joining the vertex and the mid point of the base is perpendicular to the base.

2. Write converses of the following statements.

(i) The alternate angles formed by two parallel lines and their transversal are congruent.

(ii) If a pair of the interior angles made by a transversal of two lines are supplementary then the lines are parallel.

(iii) The diagonals of a rectangle are congruent.

Problem set 1

1. Select the correct alternative from the answers of the questions given below.

(i) How many mid points does a segment have ?

(A) only one (B) two (C) three (D) many

(ii) How many points are there in the intersection of two distinct lines ?

(A) infinite (B) two (C) one (D) not a single

(iii) How many lines are determined by three distinct points?

(A) two (B) three (C) one or three (D) six

(iv) Find d(A, B), if co-ordinates of A and B are – 2 and 5 respectively.

(A) – 2 (B) 5 (C) 7 (D) 3

(v) If P - Q - R and d(P,Q) = 2, d(P,R) = 10, then find d(Q,R).

(A) 12 (B) 8 (C) $\sqrt{96}$ (D) 20

2. On a number line, co-ordinates of P, Q, R are 3, – 5 and 6 respectively. State with reason whether the following statements are true or false.

(i) d(P,Q) + d(Q,R) = d(P,R)

(ii) d(P,R) + d(R,Q) = d(P,Q)

(iii) d(R,P) + d(P,Q) = d(R,Q)

(iv) d(P,Q) – d(P,R) = d(Q,R)

3. Co-ordinates of some pairs of points are given below. Hence find the distance between each pair.

(i) 3, 6 (ii) – 9, – 1 (iii) – 4, 5 (iv) 0, – 2

(v) $x + 3$, $x - 3$ (vi) – 25, – 47 (vii) 80, – 85

4. Co-ordinate of point P on a number line is -7. Find the co-ordinates of points on the number line which are at a distance of 8 units from point P.

5. Answer the following questions.

(i) If A - B - C and d(A,C) = 17, d(B,C) = 6.5 then d (A,B) = ?

(ii) If P - Q - R and d(P,Q) = 3.4, d(Q,R)= 5.7 then d(P,R) = ?

6. Co-ordinate of point A on a number line is 1. What are the co-ordinates of points on the number line which are at a distance of 7 units from A ?

7. Write the following statements in conditional form.

(i) Every rhombus is a square.

(ii) Angles in a linear pair are supplementary.

(iii) A triangle is a figure formed by three segments.

(iv) A number having only two divisors is called a prime number.

8. Write the converse of each of the following statements.

(i) If the sum of measures of angles in a figure is 180^0, then the figure is a triangle.

(ii) If the sum of measures of two angles is 90^0 then they are complement of each other.

(iii) If the corresponding angles formed by a transversal of two lines are congruent then the two lines are parallel.

(iv) If the sum of the digits of a number is divisible by 3 then the number is divisible by 3.

9. Write the antecedent (given part) and the consequent (part to be proved) in the following statements.

(i) If all sides of a triangle are congruent then its all angles are congruent.

(ii) The diagonals of a parallelogram bisect each other.

10*. Draw a labelled figure showing information in each of the following statements and write the antecedent and the consequent.

(i) Two equilateral triangles are similar.

(ii) If angles in a linear pair are congruent then each of them is a right angle.

(iii) If the altitudes drawn on two sides of a triangle are congruent then those two sides are congruent.