2 Real Numbers

Let's study.

- Properties of rational numbers
- Properties of irrational numbers
- Surds
- Comparison of quadratic surds
- Operations on quadratic surds
- Rationalization of quadratic surds.

Let's recall.

In previous classes we have learnt about Natural numbers, Integers and Real numbers.

N = Set of Natural numbers = {1, 2, 3, 4, ...}

W = Set of Whole numbers = {0, 1, 2, 3, 4,...}

I = Set of Integers = {..., –3, –2, –1, 0, 1, 2, 3,...}

Q = Set of Rational numbers = $\{\frac{p}{q} \mid p, q \in I, q \neq 0\}$

R = Set of Real numbers.

$N \subseteq W \subseteq I \subseteq Q \subseteq R$.

Order relation on rational numbers :

$\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers where $q > 0$, $s > 0$

(i) If $p \times s = q \times r$ then $\frac{p}{q} = \frac{r}{s}$ (ii) If $p \times s > q \times r$ then $\frac{p}{q} > \frac{r}{s}$

(iii) If $p \times s < q \times r$ then $\frac{p}{q} < \frac{r}{s}$

Let's learn.

Properties of rational numbers

If a, b, c are rational numbers then

Property	Addition	Multiplication
1. Commutative	$a + b = b + a$	$a \times b = b \times a$
2. Associative	$(a + b) + c = a + (b + c)$	$a \times (b \times c) = (a \times b) \times c$
3. Identity	$a + 0 = 0 + a = a$	$a \times 1 = 1 \times a = a$
4. Inverse	$a + (-a) = 0$	$a \times \frac{1}{a} = 1 \quad (a \neq 0)$

Let's recall.

Decimal form of any rational number is either terminating or non-terminating recurring type.

Terminating type	Non terminating recurring type
(1) $\frac{2}{5} = 0.4$	(1) $\frac{17}{36} = 0.472222... = 0.47\dot{2}$
(2) $-\frac{7}{64} = -0.109375$	(2) $\frac{33}{26} = 1.2692307692307... = 1.2\overline{692307}$
(3) $\frac{101}{8} = 12.625$	(3) $\frac{56}{37} = 1.513513513... = 1.\overline{513}$

Let's learn.

To express the recurring decimal in $\frac{p}{q}$ form.

Ex. (1) Express the recurring decimal 0.777.... in $\frac{p}{q}$ form.

Solution : Let $x = 0.777... = 0.\dot{7}$

$\therefore\ 10x = 7.777... = 7.\dot{7}$

$\therefore\ 10x - x = 7.\dot{7} - 0.\dot{7}$

$\therefore\ 9x = 7$

$\therefore\ x = \frac{7}{9}$

$\therefore\ 0.777... = \frac{7}{9}$

Ex. (2) Express the recurring decimal 7.529529529... in $\frac{p}{q}$ form.

Solution : Let $x = 7.529529... = 7.\overline{529}$

$\therefore\ 1000\ x = 7529.529529... = 7529.\overline{529}$

$\therefore\ 1000\ x - x = 7529.\overline{529} - 7.\overline{529}$

$\therefore\ 999\ x = 7522.0$ $\quad \therefore\ x = \frac{7522}{999}$

$\therefore\ 7.\overline{529} = \frac{7522}{999}$

Use your brain power!

How to convert $2.4\dot{3}$ in $\frac{p}{q}$ form ?

Remember this !

(1) Note the number of recurring digits after decimal point in the given rational number. Accordingly multiply it by 10, 100, 1000.

e.g. In $2.\dot{3}$, digit 3 is the only recurring digit after decimal point, hence to convert $2.\dot{3}$ in $\frac{p}{q}$ form multiply $2.\dot{3}$ by 10.

In $1.\overline{24}$ digits 2 and 4 both are recurring therefore multiply $1.\overline{24}$ by 100.

In $1.\overline{513}$ digits 5, 1 and 3 are recurring so multiply $1.\overline{513}$ by1000.

(2) Notice the prime factors of the denominator of a rational number. If the prime factors are 2 or 5 only then its decimal expansion is terminating. If the prime factors are other than 2 or 5 also then its decimal expansion is non terminating and recurring.

Practice set 2.1

1. Classify the decimal form of the given rational numbers into terminating and non-terminating recurring type.

   (i) $\frac{13}{5}$ (ii) $\frac{2}{11}$ (iii) $\frac{29}{16}$ (iv) $\frac{17}{125}$ (v) $\frac{11}{6}$

2. Write the following rational numbers in decimal form.

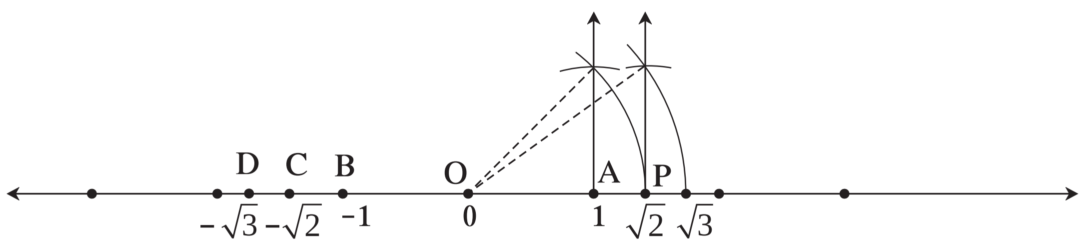
   (i) $\frac{127}{200}$ (ii) $\frac{25}{99}$ (iii) $\frac{23}{7}$ (iv) $\frac{4}{5}$ (v) $\frac{17}{8}$

3. Write the following rational numbers in $\frac{p}{q}$ form.

   (i) $0.\dot{6}$ (ii) $0.\overline{37}$ (iii) $3.\overline{17}$ (iv) $15.\overline{89}$ (v) $2.\overline{514}$

Let's recall.

The numbers $\sqrt{2}$ and $\sqrt{3}$ shown on a number line are not rational numbers means they are irrational numbers.



On a number line OA = 1 unit. Point B which is left to the point O is at a distance of 1 unit. Co-ordinate of point B is –1. Co-ordinate of point P is $\sqrt{2}$ and its opposite number $-\sqrt{2}$ is shown by point C. The co-ordinate of point C is $-\sqrt{2}$. Similarly, opposite of $\sqrt{3}$ is $-\sqrt{3}$ which is the co-ordinate of point D.

Let's learn.

Irrational and real numbers

$\sqrt{2}$ is irrational number. This can be proved using indirect proof.

Let us assume that $\sqrt{2}$ is rational. So $\sqrt{2}$ can be expressed in $\frac{p}{q}$ form.

$\frac{p}{q}$ is the reduced form of rational number hence p and q have no common factor other than 1.

$\sqrt{2} = \frac{p}{q}$ $\qquad\qquad$ $\therefore \quad 2 = \frac{P^2}{q^2}$ (Squaring both the sides)

$\therefore \quad 2q^2 = p^2$

$\therefore \quad p^2$ is even.

$\therefore \quad p$ is also even means 2 is a factor of p. ....(I)

$\therefore \quad p = 2t$ $\qquad$ $\therefore \; p^2 = 4t^2$ $\qquad$ $t \in I$

$\therefore \quad 2q^2 = 4t^2$ ($\because \; p^2 = 2q^2$) $\quad \therefore \; q^2 = 2t^2$ $\quad \therefore \; q^2$ is even. $\therefore \; q$ is even.

$\therefore \quad$ 2 is a factor of q. .... (II)

From the statement (I) and (II), 2 is a common factor of p and q both.

This is contradictory because in $\frac{p}{q}$; we have assumed that p and q have no common factor except 1.

$\therefore \quad$ Our assumption that $\sqrt{2}$ is rational is wrong.

$\therefore \quad \sqrt{2}$ is irrational number.

Similarly, one can prove that $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.

Numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ can be shown on a number line.

The numbers which are represented by points on a number line are real numbers.

In a nutshell, **Every point on a number line is associated with a unique a 'Real number' and every real number is associated with a unique point on the number line.**

We know that every rational number is a real number. But $\sqrt{2}$, $\sqrt{3}$, $-\sqrt{2}$, π, $3 + \sqrt{2}$ etc. are not rational numbers. It means that **Every real number may not be a rational number**.

Decimal form of irrational numbers

Find the square root of 2 and 3 using devision method.

Square root of 2

	1.41421...
1	$2.\overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$ ....
+1	−1
24	100
+4	−96
281	400
+ 1	−281
2824	11900
+ 4	−11296
28282	60400
+ 2	− 56564
28284 1	0383600

$\therefore\ \sqrt{2} = 1.41421...$

Square root of 3

	1.732....
1	$3.\ \overline{00}\ \overline{00}\ \overline{00}\ \overline{00}$ ....
+1	−1
27	200
+7	−189
343	1100
+ 3	−1029
3462	007100
+ 2	−6924
3464	0176

$\therefore\ \sqrt{3} = 1.732...$

Note that in the above division, numbers after decimal point are unending, means it is non-terminating. Even no group of numbers or a single number is repeating in its quotient. So decimal expansion of such numbers is non terminating, non-recurring.

$\sqrt{2}, \sqrt{3}$ are irrational numbers hence 1.4142... and 1.732... are irrational numbers too. Moreover, **a number whose decimal expansion is non-terminating, non-recurring is irrational.**

Number π

Activity I

Draw three or four circles of different radii on a card board. Cut these circles. Take a thread and measure the length of circumference and diameter of each of the circles. Note down the readings in the given table.

No.	radius (r)	diameter (d)	circumference (c)	Ratio = $\frac{c}{d}$
1	7 cm			
2	8 cm			
3	5.5 cm			

From table the ratio $\frac{c}{d}$ is nearly 3.1 which is constant. This ratio is denoted by π (pi).

Activity II

To find the approximate value of π, take the wire of length 11 cm, 22 cm and 33 cm each. Make a circle from the wire. Measure the diameter and complete the following table.

Circle No.	Circumference (c)	Diameter (d)	Ratio of (c) to (d)
1	11 cm		
2	22 cm		
3	33 cm		

Verify ratio of circumference to the diameter of a circle is approximately $\frac{22}{7}$.

Ratio of the circumference of a circle to its diameter is constant number which is irrational. This constant number is represented by the symbol π. So the approximate value of π is $\frac{22}{7}$ or 3.14.

The great Indian mathematician Aryabhat in 499 CE has calculated the value of π as $\frac{62832}{20000} = 3.1416$.

We know that, $\sqrt{3}$ is an irrational number because its decimal expansion is non-terminating, non-recurring. Now let us see whether $2+\sqrt{3}$ is irrational or not ?

Let us assume that, $2+\sqrt{3}$ is not an irrational number.

If $2+\sqrt{3}$ is rational then let $2+\sqrt{3} = \frac{p}{q}$. $\therefore$ We get $\sqrt{3} = \frac{p}{q} - 2$.

In this equation left side is an irrational number and right side rational number, which is contradictory, so $2+\sqrt{3}$ is not a rational but it is an irrational number.

Similarly it can be proved that $2\sqrt{3}$ is irrational. The sum of two irrational numbers can be rational or irrational. Let us verify it for different numbers.

i.e., $2+\sqrt{3}+(-\sqrt{3}) = 2$, $4\sqrt{5} \div \sqrt{5} = 4$, $(3+\sqrt{5}) - (\sqrt{5}) = 3$,

$2\sqrt{3} \times \sqrt{3} = 6$ $\sqrt{2} \times \sqrt{5} = \sqrt{10}$, $2\sqrt{5} - \sqrt{5} = \sqrt{5}$

Remember this !

Properties of irrational numbers

(1) Addition or subtraction of a rational number with irrational number is an irrational number.

(2) Multiplication or division of non zero rational number with irrational number is also an irrational number.

(3) Addition, subtraction, multiplication and division of two irrational numbers can be either a rational or irrational number.

Let's learn.

Properties of order relation on Real numbers

1. If a and b are two real numbers then only one of the relations holds good. i.e. $a = b$ or $a < b$ or $a > b$
2. If $a < b$ and $b < c$ then $a < c$
3. If $a < b$ then $a + c < b + c$
4. If $a < b$ and $c > 0$ then $ac < bc$ and If $c < 0$ then $ac > bc$

Verify the above properties using rational and irrational numbers.

Square root of a Negative number

We know that, if $\sqrt{a} = b$ then $b^2 = a$.

Hence if $\sqrt{5} = x$ then $x^2 = 5$.

Similarly we know that square of any real number is always non-negative. It means that square of any real number is never negative. But $(\sqrt{-5})^2 = -5$ $\therefore$ $\sqrt{-5}$ is not a real number.

Hence square root of a negative real number is not a real number.

Practice set 2.2

(1) Show that $4\sqrt{2}$ is an irrational number.

(2) Prove that $3 + \sqrt{5}$ is an irrational number.

(3) Represent the numbers $\sqrt{5}$ and $\sqrt{10}$ on a number line.

(4) Write any three rational numbers between the two numbers given below.

(i) 0.3 and –0.5 (ii) –2.3 and –2.33

(iii) 5.2 and 5.3 (iv) –4.5 and –4.6

Let's learn.

Root of positive rational number

We know that, if $x^2 = 2$ then $x = \sqrt{2}$ or $x = -\sqrt{2}$, where. $\sqrt{2}$ and $-\sqrt{2}$ are irrational numbers. This we know, $\sqrt[3]{7}$, $\sqrt[4]{8}$, these numbers are irrational numbers too.

If n is a positive integer and $x^n = a$, then x is the n[th] root of a . $x = \sqrt[5]{2}$. This root may be rational or irrational.

For example, $2^5 = 32$ $\therefore$ 2 is the 5[th] root of 32, but if $x^5 = 2$ then $x = \sqrt[5]{2}$, which is an irrational number.

Surds

We know that 5 is a rational number but $\sqrt{5}$ is not rational. Square root or cube root of any real number is either rational or irrational number. Similarly n^{th} root of any number is either rational or irrational.

If n is an integer greater than 1 and if a is a positive real number and n^{th} root of a is x then it is written as $x^n = a$ or $\sqrt[n]{a} = x$

If a is a positive rational number and n^{th} root of a is x and if x is an irrational number then x is called a surd. (Surd is an irrational root.)

In a surd $\sqrt[n]{a}$ the symbol $\sqrt{\ }$ is called **radical sign,** n is the **Order of the surd** and a is called **radicand**.

(1) Let $a = 7$, $n = 3$, then $\sqrt[3]{7}$ is a surd because $\sqrt[3]{7}$ is an irrational number.

(2) Let $a = 27$, $n = 3$ then $\sqrt[3]{27}$ is not a surd because $\sqrt[3]{27} = 3$ is not an irrational number. .

(3) $\sqrt[3]{8}$ is a surd or not ?

Let $\sqrt[3]{8} = p$ $\quad$ $p^3 = 8$. $\quad$ Cube of which number is 8 ?

We know 2 is cube-root of 8 or cube of 2 is 8.

$\therefore$ $\sqrt[3]{8}$ is not a surd.

(4) Whether $\sqrt[4]{8}$ is surd or not ?

Here $a = 8$, Order of surd $n = 4$; but 4^{th} root of 8 is not a rational number.

$\therefore$ $\sqrt[4]{8}$ is an irrational number. $\therefore$ $\sqrt[4]{8}$ is a surd.

This year we are going to study surds of order 2 only, means $\sqrt{3}$, $\sqrt{7}$, $\sqrt{42}$ etc.

The surds of order 2 is called **Quadratic surd.**

Simplest form of a surd

(i) $\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\qquad$ (ii) $\sqrt{98} = \sqrt{49 \times 2} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... these type of surds are in the simplest form which cannot be simplified further.

Similar or like surds

$\sqrt{2}$, $-3\sqrt{2}$, $\frac{4}{5}\sqrt{2}$ are some like surds.

If p and q are rational numbers then $p\sqrt{a}$, $q\sqrt{a}$ are called like surds. Two surds are said to be like surds if their order is equal and radicands are equal.

$\sqrt{45}$ and $\sqrt{80}$ are the surds of order 2, so their order is equal but radicands are not same, Are these like surds? Let us simplify it as follows :

$\sqrt{45} = \sqrt{9\times5} = \sqrt{9} \times \sqrt{5} = 3\sqrt{5}$ and $\sqrt{80} = \sqrt{16\times5} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$

$\therefore$ $3\sqrt{5}$ and $4\sqrt{5}$ are now similar or like surds

means $\sqrt{45}$ and $\sqrt{80}$ are similar surds.

Remember this !

In the simplest form of the surds if order of the surds and redicand are equal then the surds are similar or like surds.

Let's learn.

Comparison of surds

Let a and b are two positive real numbers and

If $a < b$ then $a \times a < b \times a$

If $a^2 < ab$ ...(I) Similarly $ab < b^2$ ...(II)

$\therefore$ $a^2 < b^2$ ...[from (I) and (II)]

But if $a > b$ then $a^2 > b^2$ and if $a = b$ then $a^2 = b^2$

hence if $a < b$ then $a^2 < b^2$

Here a and b both are real numbers so they may be rational numbers or surds. By using above properties, let us compare the surds.

(i) $6\sqrt{2}, 5\sqrt{5}$

$\sqrt{36} \times \sqrt{2} \ \boxed{?}\ \sqrt{25} \times \sqrt{5}$

$\sqrt{72} \ \boxed{?}\ \sqrt{125}$

But $72 \ \boxed{<}\ 125$

$\therefore 6\sqrt{2} \ \boxed{<}\ 5\sqrt{5}$

Or

$(6\sqrt{2})^2 \ \boxed{\phantom{<}}\ (5\sqrt{5})^2,$

$72 < 125$

$\therefore 6\sqrt{2} \ \boxed{<}\ 5\sqrt{5}$

(ii) $8\sqrt{3}, \sqrt{192}$

$\sqrt{64} \times \sqrt{3} \ \boxed{?}\ \sqrt{192}$

$\sqrt{192} \ \boxed{?}\ \sqrt{192}$

But $192 \ \boxed{=}\ 192$

$\therefore \sqrt{192} \ \boxed{=}\ \sqrt{192}$

$\therefore 8\sqrt{3} \ \boxed{=}\ \sqrt{192}$

(iii) $7\sqrt{2}, 5\sqrt{3}$

$\sqrt{49} \times \sqrt{2} \ \boxed{?}\ \sqrt{25} \times \sqrt{3}$

$\sqrt{98} \ \boxed{?}\ \sqrt{75}$

But $98 \ \boxed{>}\ 75$

$\therefore 7\sqrt{2} \ \boxed{>}\ 5\sqrt{3}$

Or

$(7\sqrt{2})^2 \ \boxed{}\ (5\sqrt{3})^2,$

$98 > 75$

$\therefore 7\sqrt{2} \ \boxed{>}\ 5\sqrt{3}$

Operations on like surds

Mathematical operations like addition, subtraction, multiplication and division can be done on like surds.

Ex (1) Simplify : $7\sqrt{3} + 29\sqrt{3}$

Solution : $7\sqrt{3} + 29\sqrt{3} = (7 + 29)\sqrt{3} = 36\sqrt{3}$

Ex (2) Simplify : $7\sqrt{3} - 29\sqrt{3}$

Solution : $7\sqrt{3} - 29\sqrt{3} = (7 - 29)\sqrt{3} = -22\sqrt{3}$

Ex (3) Simplify : $13\sqrt{8} + \frac{1}{2}\sqrt{8} - 5\sqrt{8}$

Solution : $13\sqrt{8} + \frac{1}{2}\sqrt{8} - 5\sqrt{8} = (13 + \frac{1}{2} - 5)\sqrt{8} = (\frac{26+1-10}{2})\sqrt{8}$

$$= \frac{17}{2}\sqrt{8} = \frac{17}{2}\sqrt{4\times 2}$$

$$= \frac{17}{2} \times 2\sqrt{2} = 17\sqrt{2}$$

Ex (4) Simplify : $8\sqrt{5} + \sqrt{20} - \sqrt{125}$

Solution : $8\sqrt{5} + \sqrt{20} - \sqrt{125} = 8\sqrt{5} + \sqrt{4\times 5} - \sqrt{25\times 5}$

$$= 8\sqrt{5} + 2\sqrt{5} - 5\sqrt{5}$$

$$= (8 + 2 - 5)\sqrt{5}$$

$$= 5\sqrt{5}$$

Ex. (5) Multiply the surds $\sqrt{7} \times \sqrt{42}$.

Solution : $\sqrt{7} \times \sqrt{42} = \sqrt{7\times 42} = \sqrt{7\times 7\times 6} = 7\sqrt{6}$ ($7\sqrt{6}$ is an irrational number.)

Ex. (6) Divide the surds : $\sqrt{125} \div \sqrt{5}$

Solution : $\frac{\sqrt{125}}{\sqrt{5}} = \sqrt{\frac{125}{5}} = \sqrt{25} = 5$ (5 is a rational number.)

Ex. (7) $\sqrt{50} \times \sqrt{18} = \sqrt{25\times 2}\times\sqrt{9\times 2} = 5\sqrt{2} \times 3\sqrt{2} = 15 \times 2 = 30$

Note that product and quotient of two surds may be rational numbers which can be observed in the above Ex. 6 and Ex. 7.

Use your brain power !

$\sqrt{9+16} \stackrel{?}{=} \sqrt{9} + \sqrt{16}$

$\sqrt{100+36} \stackrel{?}{=} \sqrt{100} + \sqrt{36}$

Rationalization of surd

If the product of two surds is a rational number, each surd is called a rationalizing factor of the other surd.

Ex. (1) If surd $\sqrt{2}$ is multiplied by $\sqrt{2}$ we get $\sqrt{2\times 2} = \sqrt{4}$. $\sqrt{4} = 2$ is a rational number.

$\therefore$ $\sqrt{2}$ is rationalizing factor of $\sqrt{2}$.

Ex. (2) Multiply $\sqrt{2} \times \sqrt{8}$

$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$ is a rational number.

$\therefore$ $\sqrt{2}$ is the rationalizing factor of $\sqrt{8}$.

Similarly $8\sqrt{2}$ is also a rationalizing factor of $\sqrt{2}$.

because $\sqrt{2} \times 8\sqrt{2} = 8\sqrt{2} \times \sqrt{2} = 8 \times 2 = 16$.

$\sqrt{6}$, $\sqrt{16}$ $\sqrt{50}$ are the rationalizing factors of $\sqrt{2}$.

Remember this !

Rationalizing factor of a given surd is not unique. If a surd is a rationalizing factor of a given surd then a surd obtained by multiplying it with any non zero rational number is also a rationalizing factor of the given surd.

Ex. (3) Find the rationalizing factor of $\sqrt{27}$.

Solution : $\sqrt{27} = \sqrt{9\times 3} = 3\sqrt{3}$ $\therefore$ $3\sqrt{3} \times \sqrt{3} = 3 \times 3 = 9$ is a rational number.

$\therefore$ $\sqrt{3}$ is the rationalizing factor of $\sqrt{27}$.

Note that, $\sqrt{27} = 3\sqrt{3}$ means $3\sqrt{3} \times 3\sqrt{3} = 9 \times 3 = 27$.

Hence $3\sqrt{3}$ is also a rationalizing factor of $\sqrt{27}$. In the same way $4\sqrt{3}$, $7\sqrt{3}$, ... are also the rationalizing factors of $\sqrt{27}$. Out of all these $\sqrt{3}$ is the simplest rationalizing factor.

Ex. (4) Rationalize the denominator of $\frac{1}{\sqrt{5}}$.

Solution : $\frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ ....(multiply numerator and denominator by $\sqrt{5}$.)

Ex. (5) Rationalize the denominator of $\frac{3}{2\sqrt{7}}$.

Solution : $\frac{3}{2\sqrt{7}} = \frac{3}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{3\sqrt{7}}{2\times 7} = \frac{3\sqrt{7}}{14}$

...(multiply $2\sqrt{7}$ by $\sqrt{7}$ is sufficient to rationalize.)

Remember this !

We can make use of rationalizing factor to rationalize the denominator.
It is easy to use the numbers with rational denominator, that is why we rationalize it.

Practice set 2.3

(1) State the order of the surds given below.

(i) $\sqrt[3]{7}$ (ii) $5\sqrt{12}$ (iii) $\sqrt[4]{10}$ (iv) $\sqrt{39}$ (v) $\sqrt[3]{18}$

(2) State which of the following are surds. Justify.

(i) $\sqrt[3]{51}$ (ii) $\sqrt[4]{16}$ (iii) $\sqrt[5]{81}$ (iv) $\sqrt{256}$ (v) $\sqrt[3]{64}$ (vi) $\sqrt{\dfrac{22}{7}}$

(3) Classify the given pair of surds into like surds and unlike surds.

(i) $\sqrt{52}$, $5\sqrt{13}$ (ii) $\sqrt{68}$, $5\sqrt{3}$ (iii) $4\sqrt{18}$, $7\sqrt{2}$

(iv) $19\sqrt{12}$, $6\sqrt{3}$ (v) $5\sqrt{22}$, $7\sqrt{33}$ (vi) $5\sqrt{5}$, $\sqrt{75}$

(4) Simplify the following surds.

(i) $\sqrt{27}$ (ii) $\sqrt{50}$ (iii) $\sqrt{250}$ (iv) $\sqrt{112}$ (v) $\sqrt{168}$

(5) Compare the following pair of surds.

(i) $7\sqrt{2}$, $5\sqrt{3}$ (ii) $\sqrt{247}$, $\sqrt{274}$ (iii) $2\sqrt{7}$, $\sqrt{28}$

(iv) $5\sqrt{5}$, $7\sqrt{2}$ (v) $4\sqrt{42}$, $9\sqrt{2}$ (vi) $5\sqrt{3}$, 9 (vii) 7, $2\sqrt{5}$

(6) Simplify.

(i) $5\sqrt{3} + 8\sqrt{3}$ (ii) $9\sqrt{5} - 4\sqrt{5} + \sqrt{125}$

(iii) $7\sqrt{48} - \sqrt{27} - \sqrt{3}$ (iv) $\sqrt{7} - \dfrac{3}{5}\sqrt{7} + 2\sqrt{7}$

(7) Multiply and write the answer in the simplest form.

(i) $3\sqrt{12} \times \sqrt{18}$ (ii) $3\sqrt{12} \times 7\sqrt{15}$

(iii) $3\sqrt{8} \times \sqrt{5}$ (iv) $5\sqrt{8} \times 2\sqrt{8}$

(8) Divide, and write the answer in simplest form.

(i) $\sqrt{98} \div \sqrt{2}$ (ii) $\sqrt{125} \div \sqrt{50}$ (iii) $\sqrt{54} \div \sqrt{27}$ (iv) $\sqrt{310} \div \sqrt{5}$

(9) Rationalize the denominator.

(i) $\dfrac{3}{\sqrt{5}}$ (ii) $\dfrac{1}{\sqrt{14}}$ (iii) $\dfrac{5}{\sqrt{7}}$ (iv) $\dfrac{6}{9\sqrt{3}}$ (v) $\dfrac{11}{\sqrt{3}}$

Let's recall.

We know that,

If $a > 0$, $b > 0$ then $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$(a+b)(a-b) = a^2 - b^2$; $\left(\sqrt{a}\right)^2 = a$; $\sqrt{a^2} = a$

Multiply.

Ex. (1) $\sqrt{2}\,(\sqrt{8} + \sqrt{18})$

$= \sqrt{2\times 8} + \sqrt{2\times 18}$

$= \sqrt{16} + \sqrt{36}$

$= 4 + 6$

$= 10$

Ex. (2) $(\sqrt{3} - \sqrt{2})(2\sqrt{3} - 3\sqrt{2})$

$= \sqrt{3}\left(2\sqrt{3} - 3\sqrt{2}\right) - \sqrt{2}\left(2\sqrt{3} - 3\sqrt{2}\right)$

$= \sqrt{3}\times 2\sqrt{3} - \sqrt{3}\times 3\sqrt{2} - \sqrt{2}\times 2\sqrt{3} + \sqrt{2}\times 3\sqrt{2}$

$= 2\times 3 - 3\sqrt{6} - 2\sqrt{6} + 3\times 2$

$= 6 - 5\sqrt{6} + 6$

$= 12 - 5\sqrt{6}$

Let's learn.

Binomial quadratic surd

- $\sqrt{5} + \sqrt{3}$; $\frac{3}{4} + \sqrt{5}$ are the binomial quadratic surds form. $\sqrt{5} - \sqrt{3}$; $\frac{3}{4} - \sqrt{5}$ are also binomial quadratic surds.

Study the following examples.

- $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
- $(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3}) = (\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$
- $(\sqrt{3} + \sqrt{7})(\sqrt{3} - \sqrt{7}) = (\sqrt{3})^2 - (\sqrt{7})^2 = 3 - 7 = -4$
- $(\frac{3}{2} + \sqrt{5})(\frac{3}{2} - \sqrt{5}) = (\frac{3}{2})^2 - (\sqrt{5})^2 = \frac{9}{4} - 5 = \frac{9-20}{4} = -\frac{11}{4}$

The product of these two binomial surds $(\sqrt{5} + \sqrt{3})$ and $(\sqrt{5} - \sqrt{3})$ is a rational number, hence these are the conjugate pairs of each other.

Each binomial surds in the conjugate pair is the rationalizing factor for other.

Note that for $\sqrt{5} + \sqrt{3}$, the conjugate pair of binomial surd is $\sqrt{5} - \sqrt{3}$ or $\sqrt{3} - \sqrt{5}$.

Similarly for $7 + \sqrt{3}$, the conjugate pair is $7 - \sqrt{3}$ or $\sqrt{3} - 7$.

Remember this !

The product of conjugate pair of binomial surds is always a rational number.

Let's learn.

Rationalization of the denominator

The product of conjugate binomial surds is always a rational number - by using this property, the rationalization of the denominator in the form of binomial surd can be done.

Ex. (1) Rationalize the denominator $\dfrac{1}{\sqrt{5}-\sqrt{3}}$.

Solution : The conjugate pair of $\sqrt{5}-\sqrt{3}$ is $\sqrt{5}+\sqrt{3}$.

$$\frac{1}{\sqrt{5}-\sqrt{3}} = \frac{1}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{\sqrt{5}+\sqrt{3}}{5-3} = \frac{\sqrt{5}+\sqrt{3}}{2}$$

Ex. (2) Rationalize the denominator $\dfrac{8}{3\sqrt{2}+\sqrt{5}}$.

Solution : The conjugate pair of $3\sqrt{2}+\sqrt{5}$ is $3\sqrt{2}-\sqrt{5}$

$$\frac{8}{3\sqrt{2}+\sqrt{5}} = \frac{8}{3\sqrt{2}+\sqrt{5}} \times \frac{3\sqrt{2}-\sqrt{5}}{3\sqrt{2}-\sqrt{5}}$$

$$= \frac{8\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}\right)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{8\times 3\sqrt{2} - 8\sqrt{5}}{9\times 2 - 5} = \frac{24\sqrt{2} - 8\sqrt{5}}{18 - 5} = \frac{24\sqrt{2} - 8\sqrt{5}}{13}$$

Practice set 2.4

(1) Multiply.

(i) $\sqrt{3}(\sqrt{7}-\sqrt{3})$ (ii) $(\sqrt{5}-\sqrt{7})\sqrt{2}$ (iii) $(3\sqrt{2}-\sqrt{3})(4\sqrt{3}-\sqrt{2})$

(2) Rationalize the denominator.

(i) $\dfrac{1}{\sqrt{7}+\sqrt{2}}$ (ii) $\dfrac{3}{2\sqrt{5}-3\sqrt{2}}$ (iii) $\dfrac{4}{7+4\sqrt{3}}$ (iv) $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Let's learn.

Absolute value

If x is a real number then absolute value of x is its distance from zero on the number line which is written as $|x|$, and $|x|$ is read as Absolute Value of x or modulus of x.

If $x > 0$ then $|x| = x$ If x is positive then absolute value of x is x.

If $x = 0$ then $|x| = 0$ If x is zero then absolute value of x is zero.

If $x < 0$ then $|x| = -x$ If x is negative then its absolute value is opposite of x.

Ex. (1) $|3| = 3$, $|-3| = -(-3) = 3$, $|0| = 0$

The absolute value of any real number is never negative.

Ex. (2) Find the value.

(i) $|9-5| = |4| = 4$ (ii) $|8-13| = |-5| = 5$

(iii) $|8| - |-3| = 5$ (iv) $|8| \times |4| = 8 \times 4 = 32$

Ex. (3) Solve $|x-5| = 2$.

Solution : $|x-5| = 2$ $\therefore$ $x - 5 = +2$ or $x - 5 = -2$

$\therefore$ $x = 2 + 5$ or $x = -2+5$

$\therefore$ $x = 7$ or $x = 3$

Practice set 2.5

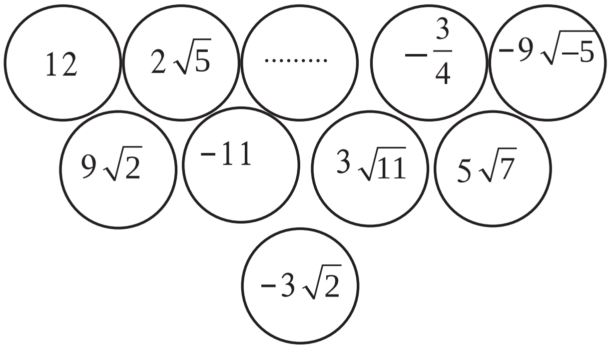
(1) Find the value.

i) $|15 - 2|$ (ii) $|4 - 9|$ (iii) $|7| \times |-4|$

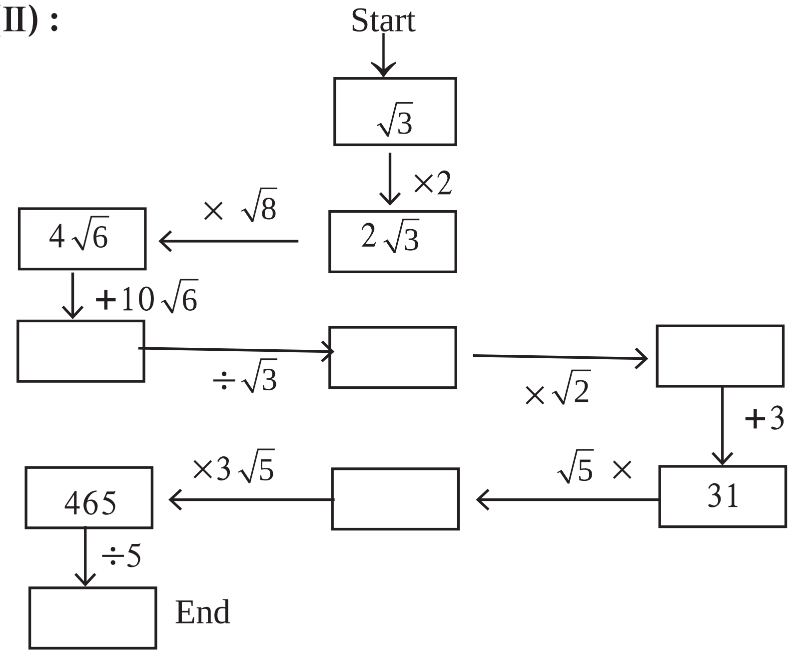
(2) Solve.

(i) $|3x-5| = 1$ (ii) $|7-2x| = 5$ (iii) $\left|\frac{8-x}{2}\right| = 5$ (iv) $\left|5+\frac{x}{4}\right| = 5$

Activity (I) : There are some real numbers written on a card sheet. Use these numbers and construct two examples each of addition, subtraction, multiplication and division. Solve these examples.



Activity (II) :



Problem set 2

(1) Choose the correct alternative answer for the questions given below.

(i) Which one of the following is an irrational number ?

(A) $\sqrt{\frac{16}{25}}$ (B) $\sqrt{5}$ (C) $\frac{3}{9}$ (D) $\sqrt{196}$

(ii) Which of the following is an irrational number?

(A) 0.17 (B) $1.\overline{513}$ (C) $0.2\overline{746}$ (D) 0.101001000.....

(iii) Decimal expansion of which of the following is non-terminating recurring ?

(A) $\frac{2}{5}$ (B) $\frac{3}{16}$ (C) $\frac{3}{11}$ (D) $\frac{137}{25}$

(iv) Every point on the number line represent, which of the following numbers?

(A) Natural numbers (B) Irrational numbers

(C) Rational numbers (D) Real numbers

(v) The number $0.\dot{4}$ in $\frac{p}{q}$ form is .....

(A) $\frac{4}{9}$ (B) $\frac{40}{9}$ (C) $\frac{3.6}{9}$ (D) $\frac{36}{9}$

(vi) What is $\sqrt{n}$, if n is not a perfect square number ?

(A) Natural number (B) Rational number

(C) Irrational number (D) Options A, B, C all are correct.

(vii) Which of the following is not a surd ?

(A) $\sqrt{7}$ (B) $\sqrt[3]{17}$ (C) $\sqrt[3]{64}$ (D) $\sqrt{193}$

(viii) What is the order of the surd $\sqrt[3]{\sqrt{5}}$?

(A) 3 (B) 2 (C) 6 (D) 5

(ix) Which one is the conjugate pair of $2\sqrt{5}+\sqrt{3}$?

(A) $-2\sqrt{5}+\sqrt{3}$ (B) $-2\sqrt{5}-\sqrt{3}$ (C) $2\sqrt{3}-\sqrt{5}$ (D) $\sqrt{3}+2\sqrt{5}$

(x) The value of $\left|12-(13+7)\times 4\right|$ is ....

(A) –68 (B) 68 (C) –32 (D) 32.

(2) Write the following numbers in $\frac{p}{q}$ form.

(i) 0.555 (ii) $29.\overline{568}$ (iii) 9.315 315 ... (iv) 357.417417... (v) $30.\overline{219}$

(3) Write the following numbers in its decimal form.

(i) $\frac{-5}{7}$ (ii) $\frac{9}{11}$ (iii) $\sqrt{5}$ (iv) $\frac{121}{13}$ (v) $\frac{29}{8}$

(4) Show that $5+\sqrt{7}$ is an irrational number.

(5) Write the following surds in simplest form.

(i) $\frac{3}{4}\sqrt{8}$ (ii) $-\frac{5}{9}\sqrt{45}$

(6) Write the simplest form of rationalising factor for the given surds.

(i) $\sqrt{32}$ (ii) $\sqrt{50}$ (iii) $\sqrt{27}$ (iv) $\frac{3}{5}\sqrt{10}$ (v) $3\sqrt{72}$ (vi) $4\sqrt{11}$

(7) Simplify.

(i) $\frac{4}{7}\sqrt{147}+\frac{3}{8}\sqrt{192}-\frac{1}{5}\sqrt{75}$ (ii) $5\sqrt{3}+2\sqrt{27}+\frac{1}{\sqrt{3}}$ (iii) $\sqrt{216}-5\sqrt{6}+\sqrt{294}-\frac{3}{\sqrt{6}}$

(iv) $4\sqrt{12}-\sqrt{75}-7\sqrt{48}$ (v*) $2\sqrt{48}-\sqrt{75}-\frac{1}{\sqrt{3}}$

(8) Rationalize the denominator.

(i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{2}{3\sqrt{7}}$ (iii) $\frac{1}{\sqrt{3}-\sqrt{2}}$ (iv) $\frac{1}{3\sqrt{5}+2\sqrt{2}}$ (v) $\frac{12}{4\sqrt{3}-\sqrt{2}}$